Practice Final Exam B for Calculus II, Math 1502, December 7, 2012

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
XI	
XII	
Total	

Name:

Section:

Name of TA:

I: (15 points) Compute with an error less than 10^{-3}

$$\int_0^1 3^{x^2}\,dx\ .$$

II: a) (7 points) Compute the limit

$$\lim_{x\to\pi/2}\left[\tan(x)+\frac{1}{x-\pi/2}\right]$$

b) b) (8 points) Does the integral

$$\int_{1/2}^{3/2}\frac{1}{x(\log x)^2}dx$$

exist?

Name:

Section:

Name of TA:

III: a) (7 points) Use alternating series theory to compute

$$\int_0^1 \cos(x^2)dx$$

with an error less than 10^{-5}.

b) (8 points) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty}(n!)^{1/n}(x-1)^n$$

IV: (15 points) Initially, a 1000 liter tank is full of brine with a concentration of 10 grams per liter. Brine with a concentration of 5 grams per liter flows into the tank at a rate of 5 liters per minute and the mixture flows out at a rate of 10 liters per minute. Find the amount of salt at time t.

Name:

Section:

Name of TA:

V: Prove or disprove (4 points each):

a) The determinant of an orthogonal matrix is ± 1.

b) Let A be an $n \times n$ matrix, then $Col(A^2) = Col(A)$.

c) If A is an $n \times n$ matrix, then $Nul(A) \subset Nul(A^2)$.

d) If for two matrices A, B, $AB = 0$ then one of the matrices must be the zero matrix.

e) If a 2×2 matrix A satisfies the equation $A^2 - 3A + 2I = 0$ then it is diagonalizable.

VI: (20 points) Given two lines,

$$\ell_1 : \vec{x}_1(s) = s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and

$$\ell_2 : \vec{x}_2(t) = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Find the distance between these two lines.

Name:

Section:

Name of TA:

VII: (10 points) Find the eigenvalues and eigenvectors of the matrix

$$B = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

VIII: (15 points) Solve the recursive relation, i.e., find a_n for arbitrary values of n,

$$a_{n+1} = 4a_n - 3a_{n-1}$$

with $a_0 = a_1 = 1$.

Name:

Section:

Name of TA:

IX: A subspace W of $\mathbb{R}^4$ is spanned by the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}$$

a) (8 points) Find the matrix that projects $\mathbb{R}^4$ onto W.

b) (7 points) Find the matrix that projects $\mathbb{R}^4$ onto $W^\perp$, the orthogonal complement of W.

Name:

Section:

Name of TA:

X: a) (10 points) Find the inverse of the matrix

$$A = \frac{1}{9}\begin{bmatrix} 1 & 8 & 4 \\ 4 & -4 & 7 \\ 8 & 1 & -4 \end{bmatrix}$$

b) (10 points) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\vec{x} = \vec{x} - 2(\vec{u} \cdot \vec{x})\vec{u}\ ,\ \ \vec{u} = \frac{1}{9}\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$$

Compute the matrix associated with this transformation and compute the eigenvalues and eigenvectors

Name:

Section:

Name of TA:

XI: (15 points) Graph the curve given by the equation

$$66x^2 - 24xy + 59y^2 = 25 \ .$$

Name:

Section:

Name of TA:

XII: (15 points) a) Show that

$$\frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

is an eigenvector for the matrix

$$A = \frac{1}{9}\begin{bmatrix} 17 & -2 & -2 \\ -2 & 14 & -4 \\ -2 & -4 & 14 \end{bmatrix}.$$

Find all the eigenvalues and eigenvectors. Find an orthonormal basis so that the matrix A is diagonal.

b) Show that

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

is an eigenvector of the matrix

$$\frac{1}{2}\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 1 & 4 \end{bmatrix}$$

Diagonalize this matrix.