Practice Test 1A for Calculus II, Math 1502, September 5, 2013

Name:

Section:

#### Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean  $\sqrt{2}$  do not write 1.414.... Show your work, otherwise credit cannot be given.

Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

## Section:

# Name of TA:

**I:** (25 points) Calculate the limits: a)

$$\lim_{x \to 0} \frac{f(x)}{f^{-1}(x)} \; ,$$

where f(x) is a differentiable and invertible function with f(0) = 0 and f'(0) = 4.

b)

$$\lim_{x \to 0} \frac{x - \int_0^x [\cos(t)]^2 dt}{x^3}$$

c)

$$\lim_{x \to 0} \left( \frac{1}{\sin(2x)} - \frac{1}{\tan(2x)} \right)$$

### Section:

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**II**:(25 points) a) Decide which of the following improper integrals exists and compute its values if it exists:

a) 
$$\int_0^\infty e^{-x} \cos(x) dx , \quad b) \int_0^\infty \frac{x}{1+x^2} dx$$

Use the comparison test to decide which of the following integrals exists:

c) 
$$\int_0^\infty \frac{1}{[\sin(x)]^2 + x^2} dx$$
,  $d \int_0^\infty \frac{x^2}{\sqrt{1 + x^6}} dx$ 

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**III:** (25 points) Which of the following series is convergent or divergent. Reason carefully!

a)

$$\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2}$$

b)

$$\sum_{k=0}^{\infty} \frac{1}{(k+2)(k+3)} \; .$$

c) Consider the convergent series

$$L = \sum_{k=0}^{\infty} \frac{1}{3^k}$$

Find the smallest n so that  $0 < L - s_n < 10^{-3}$ .

#### Section:

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**IV:** a) Solve the differential equation

$$\frac{dx}{dt} = \sin x$$

with initial condition  $x(0) = \pi/2$ .

b) At a certain moment, a tank contains 100 liters of brine with a concentration 40 grams of salt per liter. The brine is continuously drawn off at a rate of 10 liters per minute and replaced by brine containing 20 grams salt per liter. Find the amount of salt in the tank at time t later.