Practice Test 1B for Calculus II, Math 1502, September 6, 2013

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (25 points) a) Consider the recursive sequence $a_{n+1} = \sqrt{2 + a_n}, n = 0, 1, 2 \ldots$ and $a_0 = 0$. Assuming that the sequence converges, compute its limit.

b) Compute the limit $\lim_{n\to\infty} a_n$ where

$$a_n = \frac{1}{\sqrt{n^2 - 1} - \sqrt{n^2 + n}} \, .$$

c) Express the number $0.\overline{123} = 0.123123\cdots$ as a ratio of two integers.

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II: (25 points) a) For what a does the limit

$$\lim_{x\to 0} \frac{\cos(x^2)-1}{x^a}$$

exist and is not zero?

Use any test to decide which of the following integrals exists:

$$a) \quad \int_0^\infty \frac{1}{x+(x-1)^2}dx \ , \qquad b) \quad \int_{1/2}^{3/2} \frac{1}{x(\ln x)^2}dx$$

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III: (25 points) a) Solve the initial value problem

$$y' + 3x^2y = x^2 \quad y(1) = 2$$

b) (from Thomas) An aluminum beam was brought in from the outside cold into a machine shop where the temperature was held at 65° F. After 10 minutes, the beam warmed to 35° F and after another 10 minutes to 50° F. Use Newton's law of cooling to compute the initial temperature of the beam.

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IV: (25 points)

a) Consider the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^a}$$

where $a > 0$. For which values of a is this series convergent and for which ones divergent.

b) Does the series

$$\sum_{k=0}^{\infty} \sqrt{\frac{n+1}{n^3+2}}\,,$$

converge?

c) Find n so that the partial sum $s_n = \sum_{k=1}^{n} \frac{1}{k^4}$ estimates the value of the series $\sum_{k=1}^{\infty} \frac{1}{k^4}$ with an error of at most 10^{-6}.