Quiz 1, Math 1502 E1-E5, September 11, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) a) (5 points) Estimate the error for the Trapezoid rule applied to

$$\int_0^2 x\sin(x)dx$$

with $N = 10$. Your bounds on the derivatives should be rigorous.

The first and second derivatives of the integrand are

$$\sin(x) + x\cos(x) \ , \ 2\cos(x) - x\sin(x) \ .$$

Now for $x \in [0, 2]$,

$$|2\cos(x) - x\sin(x)| \le 2|\cos(x)| + x\sin(x) \le 4 \ .$$

Therfore the error is bounded by

$$\frac{4}{12}\frac{2^3}{100} = \frac{8}{300} \ .$$

b) (5 points) Solve the initial value problem

$$y' = -y^2 \ , y(0) = 1 \ .$$

$$-\frac{y'}{y^2} = 1$$

which upon integration yields

$$\frac{1}{y} = x + C \ .$$

Therefore

$$y(x) = \frac{1}{x+C}$$

which, together with the initial condition implies that

$$y(x) = \frac{1}{x+1} \ .$$

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Problem 2 (10 points) a) (4 points) Find the series for

$$\frac{x^3}{1-x^2}$$

$$x^3 \sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty} x^{2n+3}$$

b) Do the following expressions converge? (you must explain why).

i) (3 points)

$$\sum_{n=1}^{\infty} \frac{n^n}{3^{n\ln(n)}}$$

The root test yields

$$\lim_{n\to\infty} \frac{n}{3^{\ln(n)}} = \lim_{n\to\infty} \frac{e^{\ln(n)}}{3^{\ln(n)}} = \lim_{n\to\infty} (\frac{e}{3})^{\ln(n)} = 0$$

Therefore the series converges.

ii) (3 points)

$$\int_0^1 \frac{\sin(x)}{x^2} dx$$

By the limit comparison test

$$\lim_{x\to 0} \frac{\frac{\sin(x)}{x^2}}{\frac{1}{x}} = \lim_{x\to 0} \frac{x\sin(x)}{x^2} = 1$$

and therefore our integral converges if and only if $\int_0^1 1/x dx$ converges which is not the case. Hence the integral diverges.

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Problem 3 (10 points) a) (5 points) Sum the series

$$\sum_{n=3}^{\infty} \frac{3^{n+2}}{5^{n-2}}$$

Give an exact answer!

The sum can be written as

$$\sum_{n=3}^{\infty} \frac{3^{n+2}}{5^{n-2}} = 9 \cdot 25(-1 - \frac{3}{5} - \frac{3^2}{5^2} + \sum_{n=0}^{\infty} (\frac{3}{5})^n)$$

which, using the formula for the geometric series ($3/5 < 1!$) yields

$$9 \cdot 25(-\frac{25+15+9}{25} + \frac{1}{1-3/5}) \ .$$

Although not necessary, this can be simplified to

$$= 9(-49 + \frac{125}{2}) = \frac{3^5}{2}$$

b) (5 points) Sum the series

$$\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$$

Give an exact answer!

$$\frac{2}{n^2+2n} = \frac{2}{(n+2)n} = \frac{1}{n} - \frac{1}{n+2}$$

Thus the sum is a telescoping sum and

$$\sum_{n=1}^{N} \frac{2}{n^2+2n} = \sum_{n=1}^{N} \frac{1}{n} - \frac{1}{n+2} = 1 + \frac{1}{2} - \frac{1}{N+1} - \frac{1}{N+2}$$

which in the limit as $N \to \infty$ yields $3/2$.