Quiz 2, Math 1502 E1-E5, October 2, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) Compute the integral

$$\int_0^1 e^{x^4} dx$$

by a Taylor polynomial to an accuracy of 10^{-4}.

Taylors formula yields

$$e^{x^4} = \sum_{n=0}^{N} \frac{x^{4n}}{n!} + e^c \frac{x^{4N+4}}{(N+1)!}$$

where $0 \le c \le x^4$. Integrating this yields

$$\int_0^1 e^{x^4} dx = \sum_{n=0}^{N} \int_0^1 \frac{x^{4n}}{n!} dx + \int_0^1 e^c \frac{x^{4N+4}}{(N+1)!} dx \ .$$

Since $\int_0^1 x^{4n} dx = 1/(4n+1)$ we find

$$|\int_0^1 e^{x^4} dx - \sum_{n=0}^{N} \frac{1}{(4n+1)n!}| \le \int_0^1 e^c \frac{x^{4N+4}}{(N+1)!} dx$$

$$\le e \int_0^1 \frac{x^{4N+4}}{(N+1)!} dx < \frac{3}{(4N+5)(N+1)!}$$

If $N = 6$

$$\frac{3}{(4N+5)(N+1)!} < 0.000021 \ .$$

Hence

$$|\int_0^1 e^{x^4} dx - \sum_{n=0}^{6} \frac{1}{(4n+1)n!}| < 0.000021 \ .$$

Continuation of Problem 1

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Problem 2 (10 points) a) (5 points) Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{1}{n^2 5^n}(x-1)^n$$

Ratio test leads to convergence provided that

$$\frac{|x-1|}{5} < 1$$

Hence radius of convergence is 5. At $x = -4$ and $x = 6$ the series converges. Interval of convergence is $[-4, 6]$.

b) (5 points) Find the limit as $x \to 0$ of

$$\frac{e^{x^2} - 1}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} = 1 \ .$$

Continuation of Problem 2

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Problem 3 (10 points) a) (5 points) Find the point on the line $(1, 2, 1) + t\langle 1, 2, 3\rangle$ that is closest to the point $(5, 8, 5)$.

Consider any plane perpendicular to the line passing through P. It has the equation

$$x + 2y + 3z = 36$$

since P has to be on the plane. Intersecting this plane with the line yields

$$(1 + t) + 2(2 + 2t) + 3(1 + 3t) = 36$$

or $t = 2$. hence the point of intersection is

$$(1, 2, 1) + 2\langle 1, 2, 3\rangle = (3, 6, 7) \ .$$

b) (5 points) Compute the area of the parallelogram spanned by the vectors $\langle 1, 2, 3\rangle$ and $\langle 3, 2, 1\rangle$.

$$\langle 1, 2, 3\rangle \times \langle 3, 2, 1\rangle = \langle -4, 8, -4\rangle$$

and the area is $4\sqrt{6}$.

Extra credit (5 points) Find the point on the sphere $(x - 1)^2 + (y - 2)^2 + (z - 2)^2 = 9$ that is farthest away from the origin. (Geometry!)

The point must be on the line that passes through the center and the origin and hence it is of the form $t(1, 2, 2)$ for some t. To be on the sphere we must have that

$$(t - 1)^2 + (2t - 2)^2 + (2t - 2)^2 = 9 \ ,$$

which yields $(t - 1)^2[1 + 4 + 4] = 9$ or $t - 1 = \pm 1$. Thus there are two solution $t = 0$ which is the origin and $t = 2$ which is the point farthest away: $(2, 4, 4)$.

Continuation of Problem 3