Quiz 3 Solutions, Math 1502 E1-E5, October 23, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) a) Find all the solutions of

$$\begin{aligned} x + 2y - z &= 7 \\ 3x - y + 11z &= 21 \\ 2x + 7y - 8z &= 14 \end{aligned}$$

Row reducing

$$\begin{bmatrix} 1 & 2 & -1 & 7 \\ 3 & -1 & 11 & 21 \\ 2 & 7 & -8 & 14 \end{bmatrix}$$

leads to

$$\begin{bmatrix} 1 & 2 & -1 & 7 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the solutions are given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

where z is an arbitrary number.

b) (3 points) Find two vectors in $\mathbb{R}^3$ that span the plane $x - 4y + 2z = 0$.

Solve $x = 4y - 2z$, where y, z are arbitrary numbers. Hence

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4y - 2z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Continuation of Problem 1

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Problem 2 (10 points) For which values of a are the vectors

$$\begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} a \\ 6 \\ -5 \end{bmatrix}$$

linearly independent?

We row reduce

$$\begin{bmatrix} 4 & 3 & a \\ 2 & 0 & 6 \\ 1 & 4 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -5 \\ 2 & 0 & 6 \\ 4 & 3 & a \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -5 \\ 0 & -8 & 16 \\ 0 & -13 & a+20 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & -5 \\ 0 & -8 & 16 \\ 0 & 0 & a-6 \end{bmatrix}$$

If $a \neq 6$ then the vectors are linearly independent.

Continuation of Problem 2

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Problem 3 (10 points) Find the matrix A associated with the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 2x_1 - 3x_2 \\ 3x_1 \\ x_1 - x_2 \end{bmatrix} ,$$

i.e., find a matrix A so that $T(\vec{x}) = A\vec{x}$.

$$A = \begin{bmatrix} 2 & -3 \\ 3 & 0 \\ 1 & -1 \end{bmatrix}$$

b) (5 points) Is the linear transformation $T(\vec{x}) = \begin{bmatrix} x_1 + 2x_2 \\ x_1 - x_2 \end{bmatrix}$ onto?

$$\begin{bmatrix} 1 & 2 & b_1 \\ 1 & -1 & b_2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & b_1 \\ 0 & -3 & b_2 - b_1 \end{bmatrix}$$

Is consistent for every vector $\vec{b}$ and hence T is onto.

Extra credit (5 points) A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ to the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the matrix associated with T, i.e., the matrix A so that $T(\vec{x}) = A\vec{x}$.

Because

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and the matrix is

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Continuation of Problem 3