Quiz 4, Math 1502 E1-E5, November 13, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

a) (4 points) Find a basis for the Column Space $Col(A)$.
Row reduction leads to

$$\begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The first and third column are pivot columns and hence

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

are a basis for $Col(A)$.
b) (6 points) Find a basis for the Null Space, $Nul(A)$. The variables x_1 and x_4 are free and

$$x_3 = 2x_4 \ , x_1 = -3x_2 - 4x_3 - 2x_4 = -3x_2 - 10x_4 \ ,$$

so that

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -10 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

and the two vectors are a basis for $Nul(A)$.

Continuation of Problem 1

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Problem 2 (10 points) a) (5 points) Consider the matrix A and the vectors $\vec{u}$ and $\vec{v}$ given by

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 3 & 1 \\ 5 & 5 & 2 \end{bmatrix}, \vec{u} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Which one of the two is an eigenvector of A and what is the corresponding eigenvalue.

$$A\vec{u} = \begin{bmatrix} 4 \\ 20 \\ 24 \end{bmatrix} \neq \lambda\vec{u}$$

for any λ.

$$A\vec{v} = \begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix} = 7\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 7\vec{v}$$

Hence $\vec{v}$ is an eigenvector with eigenvalue 7.

b) Find the volume of the parallelepiped spanned by the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

Compute

$$\det\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = 1 \cdot \det\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - 1 \cdot \det\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = -1 - 1 = -2 .$$

Hence the volume is 2 units.

Continuation of Problem 2

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Problem 3 (10 points) Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, and for each of the eigenvalues find a basis for the corresponding eigenspace.

One eigenvalue is 1 with the corresponding eigenvector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$\det(A - \lambda I) = (1 - \lambda)(\lambda - 2)^2$$

and the other eigenvalue is 2, a two fold eigenvalue. Row reducing

$$A - 2I = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

This yields $x_2 = x_3$ and $x_1 = 2x_2 + 3x_3 = 5x_3$ Hence we have a single eigenvector as basis vector

$$\begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} .$$

Extra credit (5 points) Find a 2×2 matrix whose Column Space is the line $x_1 + 3x_2$ and whose Null Space is the line $x_1 = x_2$.

The vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is in the Nullspace of the matrix. Hence it must be of the form

$$\begin{bmatrix} a & -a \\ b & -b \end{bmatrix}$$

The column space is one dimensional and hence the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ must be on the line $x_1 + 3x_2 = 0$. Hence $a = -3b$ and the matrix is

$$b \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$$

where $b \neq 0$.

Continuation of Problem 3