1. Prepquiz 1A Solutions

Problem 1: Consider the trajectories of two particles, the first moves according to

$$\vec{r}_1(t) = (-5t^2 + 50t)\vec{k} + t\vec{i}$$

where t denotes time and the second moves according to

$$\vec{r}_2(t) = (-5t^2 + 50t)\vec{k} + t\vec{i} + 2\vec{i}$$

a) Is there a time where these two particles meet, if yes, when?

We have to find t so that $\vec{r}_1(t) = \vec{r}_2(t)$. This equation reduces to $2\vec{i} = 0$ which is not possible. So the two particles never meet.

b) Do the two curves described by these two motions intersect and if yes, where?

To check this we have to find a time t and a time s so that $\vec{r}_1(t) = \vec{r}_2(s)$. This amounts to solving the equations

$$-5t^2 + 50t = -5s^2 + 50s \ , \ t = s + 2 \ .$$

The results are $s = 4$ and $t = 6$. Hence the curves intersect at the point $120\vec{k} + 6\vec{i}$

Problem 2: A curve is given in terms of

$$\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + e^t\vec{k} \ ,$$

where $0 \le t \le 2\pi$. Compute the curvature of the curve at every point.

We may use the formula

$$\kappa = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3} \ .$$

We have

$$\vec{v} = -\sin t\vec{i} + \cos t\vec{j} + e^t\vec{k} \ , |\vec{v}| = \sqrt{1 + e^{2t}}$$

and

$$\vec{a} = -\cos t\vec{i} - \sin t\vec{j} + e^t\vec{k}$$

so that

$$\vec{v} \times \vec{a} = (\cos t + \sin t)e^t\vec{i} + (-\cos t + \sin t)e^t\vec{j} + \vec{k} \ .$$

and

$$\kappa = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3} = \frac{\sqrt{1 + 2e^{2t}}}{(1 + e^{2t})^{3/2}}$$

Problem 3: Sketch the quadrics given by the equations

$$a) \ 4x^2 + \frac{y^2}{4} + z^2 = 1 \ ,$$

This is an ellipsoid containing the points $(\pm\frac{1}{2}, 0, 0)$, $(0, \pm 2, 0)$, $(0, 0, \pm 1)$. (The $\pm$ are independent.)

$$b) \ 4x^2 - \frac{y^2}{4} + z^2 = 1 \ ,$$

This is a hyperboloid. A connected surface whose cross sections parallel to the $x-z$ plane are ellipses of the from $4x^2+z^2=1+\frac{y^2}{4}$ The quadric

$$c)\ 4x^2-\frac{y^2}{4}-z^2=1\ .$$

describes a hyperboloid which consists of two disconnected pieces. Write the equation in the form

$$4x^2-1=\frac{y^2}{4}+z^2$$

we see that one surface starts at $x=1/2$ and the other at $x=-1/2$. For $|x|>1/2$ the sections in the $y-z$ plane are ellipses.