Quiz 1 Math 2401

Tom Morley & Michael Loss
Closed book, no notes, calculators allowed.

Name :

TA :

Problem 1 (10 points)

Find the unit tangent, and curvature of the motion curve described by:
r (t) = t^2 i + 3 t^2 j + t k

r = { t^2 , 3 t^2, t}

$\{t^2, 3\,t^2, t\}$

v = D[r, t]

$\{2\,t, 6\,t, 1\}$

speed = Sqrt[v . v]

$\sqrt{1+40\,t^2}$

normal calculation :

T = v / speed

$$\left\{\frac{2\,t}{\sqrt{1+40\,t^2}}, \frac{6\,t}{\sqrt{1+40\,t^2}}, \frac{1}{\sqrt{1+40\,t^2}}\right\}$$

KappaN = D[T, t] / speed // Simplify

$$\left\{\frac{2}{(1+40\,t^2)^2}, \frac{6}{(1+40\,t^2)^2}, -\frac{40\,t}{(1+40\,t^2)^2}\right\}$$

kappa = Sqrt[KappaN. KappaN] // Simplify

$$2\sqrt{10}\sqrt{\frac{1}{(1+40\,t^2)^3}}$$

a = D[v, t]

$\{2, 6, 0\}$

```
Sqrt [Cross[v, a] . Cross[v, a]] / speed^3
```

$$\frac{2\sqrt{10}}{\left(1+40\,t^{2}\right)^{3/2}}$$

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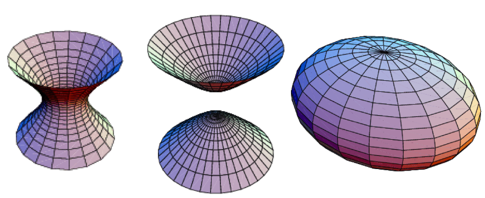
Problem 2 (10 points)

Match the following three equations (1, 2, 3) to the graphs shown (a, b, c)

1. $(x-1)^2 + \frac{y^2}{4} + \frac{z^2}{4} = 1$

2 $(x-1)^2 - \frac{y^2}{4} + \frac{z^2}{4} = 1$

3 $(x-1)^2 - \frac{y^2}{4} - \frac{z^2}{4} = 1$

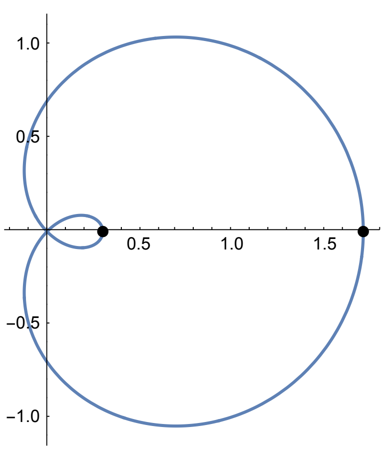


a b c

1 c, 2 b, 3 a

Let κ be the (unsigned) curvature. Mark on the following curve where is the curvature the smallest and where is it the largest.

Out[29]=



Smallest is at the right most point.

Largest is at the middle of little loop

Some computations

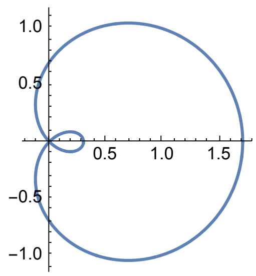
```
(x - 2)^2 - (y + 1)^2 - z^2 - 1 // ExpandAll
```

$2 - 4x + x^2 - 2y - y^2 - z^2$

Consider the curve :

```
In[1]:= p = ParametricPlot[{(.7 + Cos[t]) Cos[t], (.7 + Cos[t]) Sin[t]}, {t, 0, 2 Pi}]
```

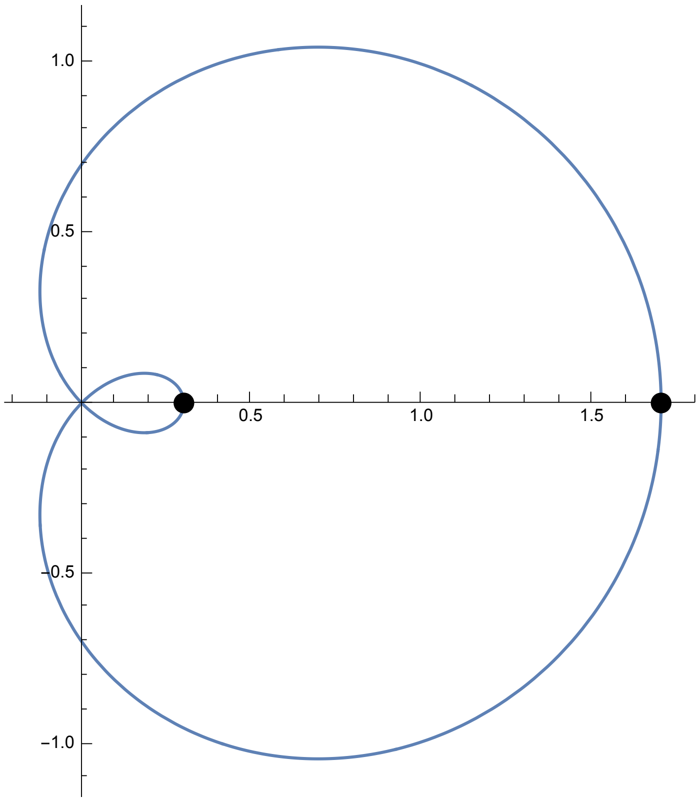
Out[1]=




```
In[2]:= sp = {(.7 + Cos[t]) Cos[t], (.7 + Cos[t]) Sin[t]} /. t → 0
Out[2]= {1.7, 0.}

In[3]:= lp = {(.7 + Cos[t]) Cos[t], (.7 + Cos[t]) Sin[t]} /. t → π
Out[3]= {0.3, 0.}

In[23]:= psp = Point[sp]
Out[23]= Point[{1.7, 0.}]

In[24]:= plp = Point[lp]
Out[24]= Point[{0.3, 0.}]
```

In[29]:=
```
Show[p, Graphics[{PointSize[.03], psp, plp}]]
```

Out[29]=



In[5]:=
```
Clear[t]
```

In[13]:=
```
r = {(.7 + Cos[t]) Cos[t], (.7 + Cos[ t]) Sin[t], 0}
```

Out[13]= {Cos[t] (0.7 + Cos[t]), (0.7 + Cos[t]) Sin[t], 0}

In[14]:=
```
v = D[r, t]
```

Out[14]= $\{-\text{Cos}[t]\,\text{Sin}[t] - (0.7 + \text{Cos}[t])\,\text{Sin}[t],\ \text{Cos}[t]\,(0.7 + \text{Cos}[t]) - \text{Sin}[t]^2,\ 0\}$

In[17]:=
```
speed = Simplify[Sqrt[v. v]]
```

Out[17]= $\sqrt{1.49 + 1.4\,\text{Cos}[t]}$

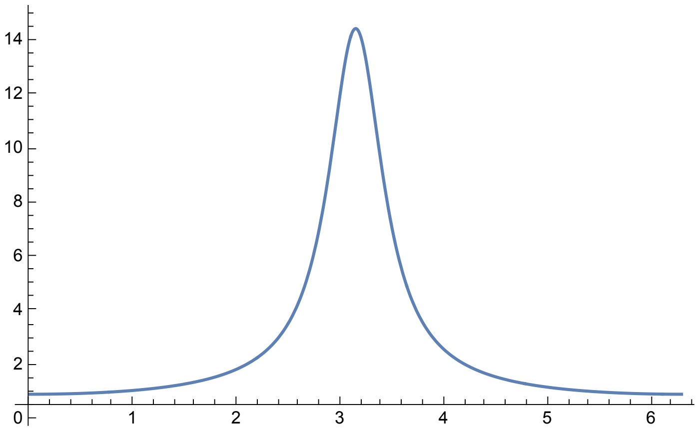
In[15]:=
```
a = Simplify[ D[v, t]]
```

Out[15]= $\{-2.\,(0.35\,\text{Cos}[t] + \text{Cos}[t]^2 - 1.\,\text{Sin}[t]^2),\ (-0.7 - 4.\,\text{Cos}[t])\,\text{Sin}[t],\ 0\}$

In[20]:=
```
kappa =
 Assuming[0 ≤ t && t ≤ 2 π, Sqrt [Cross[v, a] . Cross[v, a]] / speed^3 // Simplify]
```

Out[20]= $(2.49 + 2.1\,\text{Cos}[t] + 1.11022\times10^{-16}\,\text{Cos}[t]^2 - 1.11022\times10^{-16}\,\text{Sin}[t]^2) / (1.49 + 1.4\,\text{Cos}[t])^{3/2}$

In[22]:=
```
Plot[kappa, {t, 0, 2 Pi}, PlotRange → All]
```

Out[22]=


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Problem 3 (10 points)

Find the tangential and normal **components** of the acceleration of the motion
r (t) = 3 sin(t) i + 4 t j - 3 cos(t) k

```
In[31]:= r = {3 Sin[t] , 4 t , -3 Cos[t]}
Out[31]= {3 Sin[t], 4 t, -3 Cos[t]}

In[32]:= v = D[r, t]
Out[32]= {3 Cos[t], 4, 3 Sin[t]}

In[33]:= a = D[v, t]
Out[33]= {-3 Sin[t], 0, 3 Cos[t]}

In[34]:= speed = Sqrt[v . v] // Simplify
Out[34]= 5

In[35]:= aT = D[speed, t]
Out[35]= 0

In[36]:= vCa = Cross[v, a]
Out[36]= {12 Cos[t], -9 Cos[t]^2 - 9 Sin[t]^2, 12 Sin[t]}

In[41]:= aN = Sqrt[vCa . vCa] / speed // Simplify
Out[41]= 3

In[42]:= (* another way to compute *)
aN = Sqrt[ a . a - aT^2] // Simplify
Out[42]= 3
```

Extra Credit : (5 points)

Let C be the curve given by the intersection of
$x^2 + y^2 + z^2 = 9$
and
$x + y + z = 0$.

Find the length of C.

This is a great circle on a sphere of radius 3, so it is a cicle of radius 3, so the length is 6 π