Quiz 2 Math 2401

Tom Morley & Michael Loss

Closed book, no notes, calculators allowed.

Name :

TA :

Problem 1 (10 points)

Let $f(x, y) = x^3 y - \ln(x + y^2)$. Find an equation for the tangent plane to the surface $z = f(x,y)$ at the point $x = 0, y = 1, z = 0$.

In the (x,y) plane, find a vector that points in the direction of most rapid decrease of f at the point x = 0, y = 1.

In[44]:=
```
Clear[x, y, f]
```

In[55]:=
```
f[x_, y_] := x^3 y - Log[x + y^2]
```

As implicit, surface is g[x,y,z] = f[x, y] - z = 0

In[56]:=
```
gradg = {D[f[x, y], x], D[f[x, y], y], -1}
```

Out[56]= $\left\{3 x^2 y - \frac{1}{x + y^2}, x^3 - \frac{2 y}{x + y^2}, -1\right\}$

In[57]:=
```
gradg010 =  gradg /. {x → 0, y → 1, z → 0}
```

Out[57]= {-1, -2, -1}

In[58]:=
```
gradg010 . {x - 0, y - 1, z - 0} == 0 // Simplify
```

Out[58]= $x + 2y + z == 2$

In[47]:=
```
f[0, 1]
```

Out[47]= 0

In[60]:=
```
gradf = {D[f[x, y], x], D[f[x, y], y]}
```

Out[60]= $\left\{3x^2 y - \frac{1}{x + y^2}, x^3 - \frac{2y}{x + y^2}\right\}$

In[61]:=
```
- gradf /. {x → 0, y → 1}
```

Out[61]= {1, 2}

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Problem 2 (10 points)

Let f (x, y) = x - 6 x y - 3 y.
Find all critical points of f inside the triangle in the second quadrant (x ≤ 0, y ≥ 0) bounded by the x and y axes, and the line
-x + y = 1.
Now find the abolute max and min of f in this triangle (including the boudary).

In[11]:= **f = x - 6 x y - 3 y**

Out[11]= x - 3 y - 6 x y

In[12]:= **gf = {D[f, x] == 0, D[f, y] == 0}**

Out[12]= {1 - 6 y == 0, -3 - 6 x == 0}

In[13]:= **Solve[gf, {x, y}]**

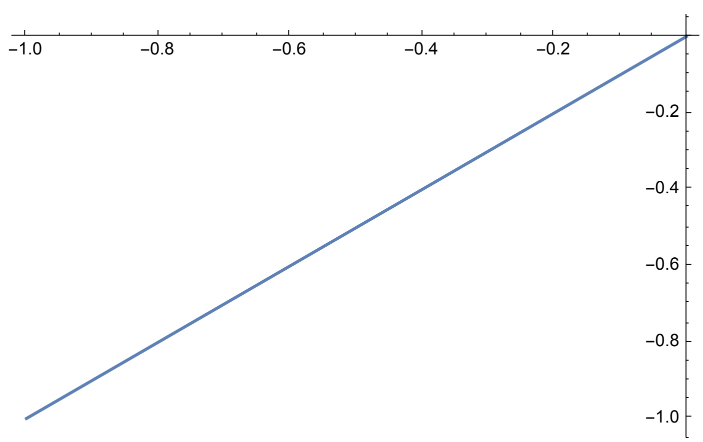
Out[13]= $\left\{\left\{x \to -\frac{1}{2}, y \to \frac{1}{6}\right\}\right\}$

In[20]:= **f1 = f /. y → 0**

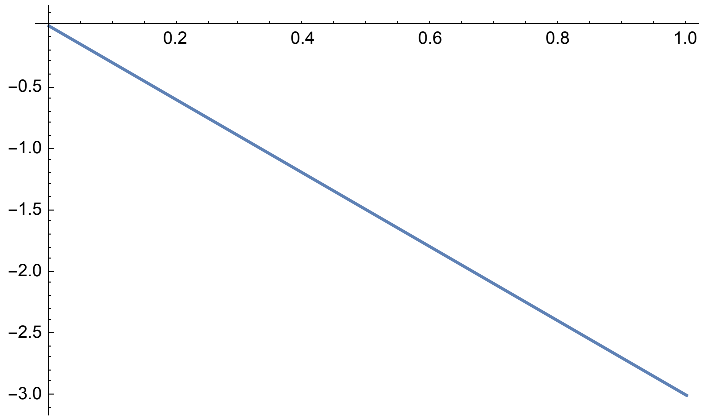
Out[20]= x

In[21]:= **Plot[f1, {x, -1, 0}]**

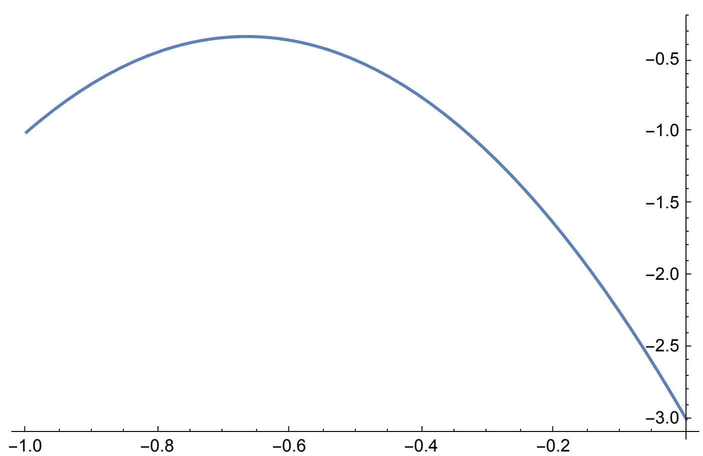
Out[21]=



In[23]:=
```
f2 = f /. x → 0
```
Out[23]= $-3\,y$

In[24]:=
```
Plot[f2, {y, 0, 1}]
```
Out[24]=


In[18]:=
```
f3 = f /. y → 1 + x // Simplify
```
Out[18]= $-3 - 8\,x - 6\,x^2$

In[19]:=
```
Plot[f3, {x, -1, 0}]
```
Out[19]=


In[25]:=
```
f /. {x → 0, y → 0}
```
Out[25]= 0

In[26]:=
```
f /. {x → 0, y → 1}
```
Out[26]= -3

In[27]:= $f\ /.\ \left\{x \to -\frac{1}{2},\ y \to \frac{1}{6}\right\}$

Out[27]= $-\frac{1}{2}$

Max is at (0,0), where f = 0
Min is at (0,1) where f = - 3

(-1/2, 1/6) is neither a absolute min nor max

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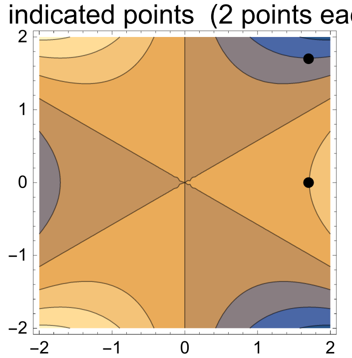
Problem 3 (10 points)

The picture is the contour plot of a function f(x,y). The lighter regions are larger values of f, while the darker regions are smaller values of f. Show on the picture the direction of the gradient of f at the two indicated points (2 points each)



Let $f(x,y) = x(x^2 - 3y^2)$. At the point (1,1), find
(2 points each):

a) A vector in the direction u for which $D_u f(1,1)$ as large as possible
b) A vector in the direction u for which $D_u f(1,1)$ as small as possible
c) A non zero vecor for which $D_u f(1,1)$ equal to zero.

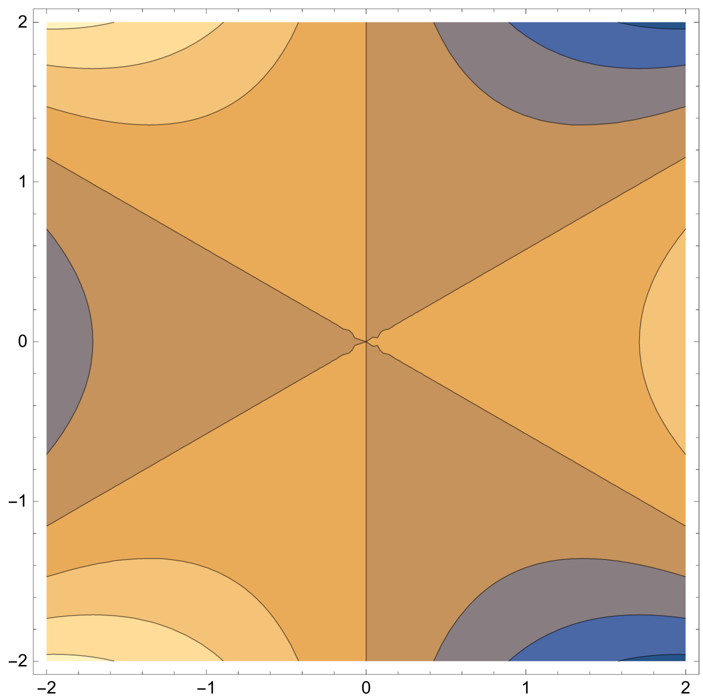
Here $D_u f(1,1)$ is the directional derivative of f at (1,1) in the direction u.

```
In[36]:= f = x (x^2 - 3 y^2)
Out[36]= x (x^2 - 3 y^2)

In[37]:= gradf = {D[f, x], D[f, y]}
Out[37]= {3 x^2 - 3 y^2, -6 x y}

In[38]:= gradf /. {x → 1.7, y → 0}
Out[38]= {8.67, 0.}
```

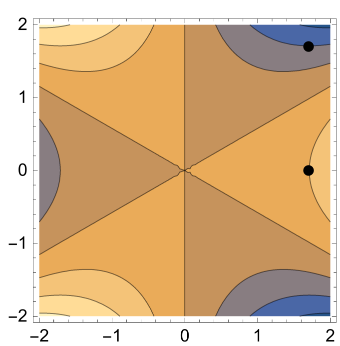
```
In[39]:= gradf /. {x → 1.7, y → 1.7}
Out[39]= {0., -17.34}
```

```
pts = Graphics[{PointSize[.035], Point[{1.7, 0}], Point[{1.7, 1.7}]}]
```

```
cp = ContourPlot[f, {x, -2, 2}, {y, -2, 2}]
```




```
Show[{cp, pts}]
```



In[40]:= `gradf11 = gradf /. {x → 1, y → 1}`

Out[40]= {0, -6}

Most uphill = - 6 j, (or just -j)
Most downhill = 6j (or just j)
Zero dirrectional derivative +/- i

Extra Credit : (5 points)

Consider the curve given by the equation
$x^3 - 3xy^2 = 8$
Find all the points on this curve where the tangent is horizontal.

In[30]:= $f = x^3 - 3 x y^2$

Out[30]= $x^3 - 3 x y^2$

In[31]:= `gf = {D[f, x] , D[f, y]}`

Out[31]= $\{3 x^2 - 3 y^2, -6 x y\}$

Tangent is horizontal where normal is vertical., So grad f must be in the y direction, that is, d[f, x] == 0

In[34]:= $\text{Solve}[\{3 x^2 - 3 y^2 == 0,\ x^3 - 3 x y^2 == 8\}, \{x, y\}]$

Out[34]= $\{\{x \to -(-2)^{2/3}, y \to -(-2)^{2/3}\}, \{x \to -(-2)^{2/3}, y \to (-2)^{2/3}\}, \{x \to -2^{2/3}, y \to -2^{2/3}\}, \{x \to -2^{2/3}, y \to 2^{2/3}\}, \{x \to (-1)^{1/3} 2^{2/3}, y \to -(-1)^{1/3} 2^{2/3}\}, \{x \to (-1)^{1/3} 2^{2/3}, y \to (-1)^{1/3} 2^{2/3}\}\}$

In[35]:= `% // N`

Out[35]= {{x → 0.793701 - 1.37473 i, y → 0.793701 - 1.37473 i},
{x → 0.793701 - 1.37473 i, y → -0.793701 + 1.37473 i}, {x → -1.5874, y → -1.5874},
{x → -1.5874, y → 1.5874}, {x → 0.793701 + 1.37473 i, y → -0.793701 - 1.37473 i},
{x → 0.793701 + 1.37473 i, y → 0.793701 + 1.37473 i}}

Answer is $(x, y) = (-2^{2/3}, -2^{2/3})$, **or** $(x, y) = (-2^{2/3}, 2^{2/3})$,