HOMEWORK 2, DUE ON TUESDAY SEPTEMBER 27.

Problem 1: Show that $\ell_1^* = \ell_\infty$.

Problem 2: Prove that any finite dimensional normed space is reflexive.

Problem 3: Show that ℓ_∞ is not separable. (Hint: Consider balls of small radii centered at sequences with integer coefficients. Show that there are uncountably many such balls.)

Problem 4: Prove that if $\{\phi_n(x)\}_{n=1}^{\infty}$ is an orthonormal basis in $L^2[a,b]$ then for all $x \in [a,b]$

$$\sum_{n=1}^{\infty} | \int_a^x \phi_n(t)dt|^2 = x - a \ .$$

(The converse also holds, but is a bit trickier to prove.)

Problem 5: Let X be a normed space and Y be a Banach Space. Show that the space of linear bounded operators $L(X \mapsto Y)$ is a Banach space.