HOMEWORK 5, DUE MARCH 1

Problem 1 (5 points): Please solve Problem 3.1.11 in 'Introduction to real analysis'.

Problem 2 (5 points): Please solve Problem 3.2.8 in 'Introduction to real analysis'.

Problem 3 (5 points): Let $\{f_n\}_{n=1}^{\infty}$ is a sequence of non-negative measurable functions on a measurable set $E \subset \mathbb{R}^d$. Prove that

$$\int_E \sum_{j=1}^{\infty} f_n = \sum_{j=1}^{\infty} \int_E f_n.$$

Problem 4 (5 points): Consider a non-negative integrable function $f : E \to \mathbb{R}_+$. where $E \subset \mathbb{R}^d$ is measurable. Show that

$$\int_E f = \int_0^{\infty} |\{x \in E : f(x) > t\}| dt$$

where the last integral is a Riemann integral.

Problem 5 (5 points): Use the result of Problem 4 to compute the Lebesgue integral

$$\int_{-1}^1 |x|^{-1/2} dx \ .$$