Problem 1 (5 points): Let $C \subset \mathbb{R}^d$ be compact and $f : C \to \mathbb{R}$ a lower semicontinuous function. Prove that f attains its minimum.

Problem 2 (8 points): recall that a function $f : [a, b] \to \mathbb{R}$ is monotone increasing if $f(x) \leq f(y)$ whenever $x, y \in [a, b]$ and $x \leq y$. The definition of monotone decreasing is similar. Prove that any monotone function is Lebesgue measurable.

Problem 3 (9 points): Let E be a measurable set in $\mathbb{R}^d$. Show that for any $\varepsilon > 0$ there exists an open set U and a closed set F such that $F \subset E \subset U$ and $|U \setminus F| < \varepsilon$.

Problem 4 (9 points): Let $E_1 \subset E_2 \subset \cdots$ be a sequence of nested measurable sets in $\mathbb{R}^d$. Prove that

$$| \cup_{k=1}^{\infty} E_k| = \lim_{n\to\infty} |E_n|$$

Problem 5 (9 points): In Egorov's theorem we had to assume that $|E| < \infty$. Give an example of a sequence of functions on the whole real line which converges but where Egorov's theorem fails.