Homework 2, due Wednesday March 1

I: Prove that if a closed symmetric operator A has a real number in its resolvent set, then it is self adjoint.

II: Let A be a self adjoint operator and B be a symmetric operator which is A bounded with bound a. Show that

$$\limsup_{\mu\to\infty} \|B(A+i\mu I)^{-1}\| \le a \ .$$

III: With the abbreviation $p = \frac{1}{i}\frac{d}{dx}$ consider the operator $A = px^5 + x^5 p$ on the domain $D(A) = C_c^\infty(\mathbb{R})$. Prove that A does not have any self adjoint extensions.

IV: Let A be a symmetric operator on a Hilbert space. Prove that the following statements are equivalent:
a) A is essentially self adjoint.
b) $\mathrm{Ker}(A \pm iI) = \{0\}$
c) $\mathrm{Ran}(A \pm iI)$ is dense.

V (Reed-Simon Vol. II): Let A be a closed symmetric operator which has a self adjoint extension. Is it possible that A has a closed symmetric extension B that has no self adjoint extension? Explain.