Homework 3, due Wednesday March 29

I: Let $\mathcal{A}$ be a commutative Banach Algebra and $\mathcal{I}$ a closed ideal. Recall that any element in the factor algebra $\mathcal{A}/\mathcal{I}$ is given as an equivalence class $[x]$ where two elements $x, y \in \mathcal{A}$ are equivalent if $x - y \in \mathcal{I}$. Also recall that

$$\|[x]\| = \inf_{z \in \mathcal{I}} \|x - z\| \ .$$

The multiplication of equivalence classes are defined by $[x][y] = [xy]$. Show that $\mathcal{A}/\mathcal{I}$ is a Banach Algebra.

II: Let $\mathcal{A}$ be a commutative Banach Algebra and $x \in \mathcal{A}$ be an element that has no inverse. Prove that x belongs to a non-trivial maximal ideal.

III: Again, $\mathcal{A}$ is a commutative Banach Algebra. Prove that closed ideal $\mathcal{I} \subset \mathcal{A}$ is a proper subset of a nontrivial ideal if and only if its factor algebra $\mathcal{A}/\mathcal{I}$ has nontrivial ideals.

IV: Show that the Gelfand transform is linear, multiplicative, and that $\widehat{e} = 1$.

V : Let e be the unit element in a C^* algebra. Proof that $e^* = e$ and that $\|e| = 1$