NAME:

QUIZ 1 FOR MATH 2551 F1-F4, AUGUST 29, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1:
a) (2 points) Find the angle between the vectors $\vec{a} = \langle 1, 2, 2\rangle$ and $\vec{b} = \langle 2, -2, 1\rangle$

$$\vec{a} \cdot \vec{b} = 0$$

angle is $\pi/2$.

b) (2 points) Find the distance between the point $(1, 1, 1)$ and the point $(2, 3, 3)$.
The difference vector is $\langle 1, 2, 2\rangle$ and its length is 3.

Problem 2: (3 points) The two planes $x + y + z = 1$ and $2x - y - z = 2$ intersect in a line. Compute this line in parametrized from.
Have to row reduce the augmented system

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & -1 & 2 \end{bmatrix}$$

which leads to

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

and $z = t$ is a free variable. Hence

$$x = 1\ , y = -t\ , z = t\ .$$

Problem 3: (3 points) Find the distance of the point $(1, 1, 1)$ to the plane $x + y + z = 0$.
Take the origin, which is a point on the plane. We have to project the vector $\vec{OP}$ onto the normal vector. Note that $\vec{OP} = \langle 1, 1, 1\rangle$ which is the same as the normal vector. Hence the projection is the vector itself and its length and hence the distance is $\sqrt{3}$.

Extra credit: (2 points) Find the distance between the point $(2, 2, 1)$ and the line

$$x = t\ , y = t\ , z = t$$

Pick any point on the line, e.g., $(0, 0, 0)$ which is the origin. Now we decompose the vector $\vec{OP} = \langle 2, 2, 1\rangle$ into a component parallel to $\vec{v} = \langle 1, 1, 1\rangle$ and a component perpendicular to $\vec{v}$. The component perpendicular is

$$\vec{v}_\perp = \vec{OP} - Proj_{\vec{v}}\vec{OP} = \vec{OP} - \frac{\vec{OP} \cdot \vec{v}}{|\vec{v}|^2}\vec{v} = \langle 2, 2, 1\rangle - \frac{5}{3}\langle 1, 1, 1\rangle$$

which equals

$$\frac{1}{3}\langle 1, 1, -2\rangle$$

and hence the distance is

$$\frac{\sqrt{6}}{3}$$