NAME:

QUIZ 3 FOR MATH 2551 F1-F4, SEPTEMBER 12, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1: a) (3 points) Find $\vec{T}$ and $\vec{N}$ for the curve given by

$$\vec{r}(t) = \langle \cos t^2, \sin t^2 \rangle \ , 0 \leq t \leq 2\pi$$

Solution:

$$\vec{r}'(t) = 2t\langle -\sin t^2, \cos t^2 \rangle$$
$$\vec{T}(t) = \langle -\sin t^2, \cos t^2 \rangle$$
$$T'(t) = -2t\langle \cos t^2, \sin t^2 \rangle$$
$$\vec{N}(t) = -\langle \cos t^2, \sin t^2 \rangle$$

Problem 2: Find the curvature $\kappa(t)$ for the curves

$$a) \text{ (1 point) } \vec{r}(t) = \langle 5 - t, 3 + t \rangle \ t \in \mathbb{R}$$

$$b) \text{ (2 points) } \vec{r}(t) = \langle t, t^2/2 \rangle \ , t \in \mathbb{R}$$

Solution: a)

$$\vec{r}'(t) = \langle -1, 1 \rangle$$
$$\vec{T}(t) = \frac{\langle -1, 1 \rangle}{\sqrt{2}}$$

which is constant and hence the curvature is zero.

b)

$$\vec{T}(t) = \frac{\langle 1, t \rangle}{\sqrt{1 + t^2}}$$

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{v}'|} = \frac{|\langle -t, 1 \rangle|}{(1 + t^2)^2} = \frac{1}{(1 + t^2)^{3/2}}$$

Problem 3: (4 points) The trajectory of a particle is given by

$$\vec{r}(t) = \langle R\cos t, R\sin t, -t^2/2 \rangle \ , \ t \in \mathbb{R}$$

where R is a positive constant. Find $a_{\vec{T}}$ and $a_{\vec{N}}$ without computing $\vec{T}$ and $\vec{N}$.

Solution: $s'(t) = |\vec{r}'(t)| = \sqrt{R^2 + t^2}$

$$a_{\vec{T}} = s''(t) = \frac{t}{\sqrt{R^2 + t^2}}$$
$$\vec{a}(t) = \vec{r}''(t) = \langle -R\cos t, -R\sin t, -1 \rangle$$

so that

$$|\vec{a}(t)| = \sqrt{R^2 + 1}$$

Thus

$$a_{\vec{N}} = \sqrt{|\vec{a}(t)|^2 - a_{\vec{T}}^2} = \sqrt{R^2 + 1 - \frac{t^2}{R^2 + t^2}} = \frac{\sqrt{R^4 + R^2 t^2 + R^2}}{\sqrt{R^2 + t^2}}$$

Extra credit: (1 point) Find $\vec{B}$ for the spiral

$$\vec{R}(t) = \langle \cos t, \sin t, t \rangle \ , t \in \mathbb{R}$$

Solution: The tangent vector is given by

$$\vec{T}(t) = \frac{\langle -\sin t, \cos t, 1 \rangle}{\sqrt{2}}$$

$$\vec{N}(t) = -\langle \cos t, \sin t, 0 \rangle$$

$$\vec{B} = \vec{T}(t) \times \vec{N}(t) = \frac{\langle \sin t, -\cos t, 1 \rangle}{\sqrt{2}}$$