TOPICS FOR PARTIAL DIFFERENTIAL EQUATIONS 2

While the first part of this course on Partial Differential Equation dealt with the main examples, the sequel is the theoretical part. The main topic will be existence and regularity of solutions for linear elliptic equations. The basic approach is to formulate the problem in such a fashion that functional analytic methods furnish the existence of "weak" solutions. E.g., reducing the existence question to the Riesz representation theorem for Hilbert spaces is one approach. A fist obstacle is the choice of spaces and for this we have to develop the considerable machinery of Sobolev Spaces. Another piece of hard work is to show regularity for this weak solutions. In order for all this to work we need an arsenal of functional inequalities. Thus the outline is:

1) Sobolev spaces, Sobolev inequalities, trace inequalities etc.

2) Weak formulation for the solutions of elliptic PDEs and existence of weak solutions.

3) Regularity theory

One very interesting topic I would like to learn is De Giorgi's method of regularity for elliptic PDEs. The method also goes under the name of De Giorgi-Nash-Moser theory, however, De Giorgi had a particular way of looking at the problem which I would like to understand myself.

Another important topic are non-linear equations. There is no theory of non-linear PDE and one has to consider specific examples, such as the non-linear Schrödinger equation or the Navier-Stokes equations. There are existence questions as well as non-existence questions and the Calculus of Variations plays a more prominent rule in this context.

Text: We shall use the book by L.C.Evans, Partial Differential Equations, Second Edition, American Mathematical Society, Providence 2010, ISBN 978-0-8218-4974-3

For other topics I will use my own notes.