

HW ASSIGNMENT #4 (DUE THURSDAY, SEPTEMBER 18)

1. (Knapp §IV.12 #12) Let G be a finite group, and let H be a normal subgroup of G of order p , where p is the smallest prime dividing $|G|$. Prove that H is contained in the center of G .
2. Determine all finite groups which have exactly two conjugacy classes.
3. If the center of a group G has index n , prove that every conjugacy class in G has at most n elements.
4. Let G be a finite group, and suppose that there is a non-identity element $x \in G$ which is conjugate to x^{-1} . Prove that $|G|$ is even.
5. Let G be a group acting transitively on the finite set A . If $|A| > 1$, show that there is some $g \in G$ having no fixed points.
6. If a finite group G has exactly n elements of order p , where p is a prime number, prove that either $n = 0$ or p divides $n + 1$.
7. Use Polya's method to solve the following problems:
 - (a) How many distinct ways are there to color the vertices of a cube using k colors?
 - (b) How many distinct ways are there to color the faces of a tetrahedron using k colors?