1. Apostol §3.13, p. 109, Exercise # 3
2. Apostol §3.13, p. 109, Exercise # 5
3. Apostol §3.17, p. 118, Exercise # 1a
4. Apostol §3.17, p. 118, Exercise # 2b
5. Apostol §3.17, p. 118, Exercise # 4

6. Let $W \subseteq \mathbb{R}^4$ be the linear span of $(1, 0, -1, 1)$ and $(2, 3, -1, 2)$. Find a basis for the orthogonal complement of $W$ (with respect to the standard inner product).

7. Let $V$ be a finite-dimensional Euclidean space, let $W$ be a subspace of $V$, and let $W^\perp$ be the orthogonal complement of $W$.
   
   (a) Prove that every vector $x \in V$ can be written uniquely as $x = y + z$ with $y \in W$ and $z \in W^\perp$.
   
   (b) Prove that $\|x\|^2 = \|y\|^2 + \|z\|^2$, and draw a picture illustrating this fact when $V = \mathbb{R}^3$ with the standard inner product.
   
   (c) Prove that $\dim(W) + \dim(W^\perp) = \dim(V)$. [Hint: If $Y = \{y_1, \ldots, y_k\}$ is an orthogonal basis for $W$ and $Z = \{z_1, \ldots, z_l\}$ is an orthogonal basis for $W^\perp$, show that $Y \cup Z = \{y_1, \ldots, y_k, z_1, \ldots, z_l\}$ is an orthogonal basis for $V$.]
   
   (d) Recall from lecture that the rank of a matrix $A$ equals the dimension of the row space of $A$. Prove that the rank of $A$ plus the dimension of the kernel of $A$ equals the number of columns of $A$. [Hint: Use part (c).]
8. Let $V$ be a finite-dimensional Euclidean space with basis $e_1, \ldots, e_n$, and let $A$ be the $n \times n$ matrix whose $ij$th entry is $\langle e_i, e_j \rangle$.

(a) If $x = x_1e_1 + \cdots + x_ne_n \in V$, show that 

$$Ax = \begin{bmatrix} \langle x, e_1 \rangle \\ \vdots \\ \langle x, e_n \rangle \end{bmatrix}.$$ 

(b) Show that $\ker(A) = \{0\}$ (i.e., that the null space of $A$ is zero).

(c) Prove that given any scalars $c_1, \ldots, c_n \in \mathbb{R}$, there is a unique vector $x \in V$ such that $\langle x, e_i \rangle = c_i$ for $1 \leq i \leq n$. 
