1. Apostol §3.17, p.118, Exercise #8
2. Apostol §3.17, p.118, Exercise #9
3. Apostol §4.4, p.123, Exercise #5
4. Apostol §4.4, p.123, Exercise #7
5. Apostol §4.4, p.123, Exercise #11
6. Apostol §4.4, p.123, Exercise #12
7. Apostol §4.4, p.123, Exercise #23
8. Does there exist a linear transformation $T$ from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that

$$T \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad T \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \text{and} \quad T \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}?$$

(Be sure to justify your answer.)

9. Find the least squares regression line for the data points

$(0, 10), (2, 6), (3, 7), (4, 6), (5, 3), (8, 1)$. 

10. Let

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \\ 1 & 1 \end{pmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$ 

(a) Find the orthogonal projection of $b$ onto the range of $A$.

(b) Find the best approximate (least squares) solution to the overdetermined system of equations $Ax = b$. 

1
11. Recall that mean $\bar{x}$ and standard deviation $\sigma_x$ of a collection $x_1, \ldots, x_n$ of real numbers are given by the formulas

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}, \quad \sigma_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}.$$ 

The Pearson correlation coefficient of a collection of $n \geq 2$ distinct data points $(x_1, y_1), \ldots, (x_n, y_n) \in \mathbb{R}^2$ is defined to be

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n \sigma_x \sigma_y},$$

where $\sigma_x$ (resp. $\sigma_y$) denotes the standard deviation of $x_1, \ldots, x_n$ (resp. $y_1, \ldots, y_n$) and $\bar{x}$ (resp. $\bar{y}$) denotes the mean of $x_1, \ldots, x_n$ (resp. $y_1, \ldots, y_n$). (The quantity $\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$ is called the covariance, so the Pearson correlation coefficient is just the covariance divided by the product of the standard deviations.)

(a) Prove that $-1 \leq r \leq 1$, and that the data points lie on a straight line if and only if $r = \pm 1$. [Hint: Use the Cauchy-Schwartz inequality.]

(b) Calculate the Pearson correlation coefficient for the data points in Problem 9.

12. Let $V$ be a finite-dimensional Euclidean space, let $W$ be a subspace of $V$, and let $p_W$ denote orthogonal projection onto $W$. Prove that for every $x \in V$, the length of $p_W(x)$ is less than or equal to the length of $x$, with equality if and only if $x \in W$. 

2