Math 1552, Exam 1: Integration Name and section: ____________________

1. (4.8.49,71,79) Find the indefinite integrals.

   (a) (10 points)

$$\int (7x-2)^3\,dx$$

   (b) (10 points)

$$\int \frac{1}{1+t^2}\,dt$$

   (c) (5 points)

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

Solution:

(a)

$$\int (7x-2)^3\,dx = \frac{1}{28}(7x-2)^4 + C.$$

(b)

$$\int \frac{1}{1+t^2}\,dt = \tan^{-1}(t) + C.$$

(c)

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta = -\csc\theta + C$$

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2. (25 points) (5.2.29,35) Find the limit

$$\lim_{n\to\infty} \sum_{j=0}^{n-1} \left[16 - \frac{j/n + (j+1)/n}{6}\right] (x_{j+1} - x_j)$$

where $x_j = j/n$ for $j = 0, 1, 2, \ldots, n$. Hint: Interpret the sum as a Riemann sum with midpoints for the evaluation points.

Solution: This is a limit of Riemann sums (with midpoint evaluation points) for

$$\int_0^1 \left[16 - \frac{x}{3}\right] dx = \left[16x - \frac{x^2}{6}\right]\Big|_0^1 = 16 - \frac{1}{6} = \frac{95}{6}.$$

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3. (5.3.42, 5.6.105)

   (a) (10 points) Find the area in the first quadrant under the graph of

   $$f(x) = \frac{9x^{10}}{\sqrt{1-x^{22}}}$$

   and to the left of the line $x = 1$.

   (b) (15 points) Find the area in the first quadrant bounded below by the line $y = x/4$, above left by the curve $y = 1 + \sqrt{x}$, above right by the curve $y = 2/\sqrt{x}$, and on the right by the line $x = 4$.

Solution:

(a) Make a u-substitution with $u = x^{11}$ and $du = 11x^{10}\,dx$.

$$\begin{aligned} A &= \int_0^1 \frac{9x^{10}}{\sqrt{1-x^{22}}}\,dx \\ &= \frac{9}{11}\int_0^1 \frac{1}{\sqrt{1-u^2}}\,du \\ &= \frac{9}{11}\sin^{-1} u\Big|_0^1 \\ &= \frac{9}{11}\sin^{-1}(1) \\ &= \frac{9\pi}{22}. \end{aligned}$$

(b)

$$\begin{aligned} A &= \int_0^1 \left[1 + \sqrt{x} - \frac{x}{4}\right] dx + \int_1^4 \left[\frac{2}{\sqrt{x}} - \frac{x}{4}\right] dx \\ &= \left[x + \frac{2}{3}x^{3/2} - \frac{x^2}{8}\right]\Big|_0^1 + \left[4\sqrt{x} - \frac{x^2}{8}\right]\Big|_1^4 \\ &= 1 + 2/3 - 1/8 + 8 - 2 - [4 - 1/8] \\ &= 11/3. \end{aligned}$$

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4. (25 points) (6.1.28) A solid is formed by rotating a certain area in the first quadrant about the y-axis. The area is bounded above by the line $y = 2$ and on the right by the graph of $x = e^y$. Find the volume of the solid.

Solution: We use the "disks" method:

$$\begin{aligned} V &= \int_0^2 \pi e^{2y}\,dy \\ &= \frac{\pi}{2} e^{2y}\Big|_0^2 \\ &= \frac{\pi}{2}\left[e^4 - 1\right]. \end{aligned}$$