Math 1552, Exam 2: Integration Name and section: ____________________

1. (25 points) (6.1.52) Find the volume of the solid generated by revolving the triangular region bounded by the lines $y = 2x$, $y = 0$, and $x = 1$ about the line $x = 2$.

Solution: Using the method of washers, the outer radius of the washer is $2 - y/2$ while the inner radius is 1. Thus, the volume is

$$\begin{aligned} V &= \int_0^2 \left[\pi(2 - y/2)^2 - \pi(1)^2\right] \, dz \\ &= \pi \int_0^2 (3 - 2y + y^2/4) \, dz \\ &= \pi(6 - 4 + 8/12) \\ &= 8\pi/3. \end{aligned}$$

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2. (25 points) (6.4.14) Find the surface area of the surface generated by rotating the graph of $y = \sqrt{x}$ between $x = 2$ and $x = 6$ about the x-axis.

Solution:

$$\begin{aligned} A &= \int_2^6 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx \\ &= \pi\int_2^6 \sqrt{4x+1}\,dx \\ &= \frac{\pi}{4}\frac{2}{3}(4x+1)^{3/2}\Big|_2^6 \\ &= \frac{\pi}{6}(125-27) \\ &= \frac{49\pi}{3}. \end{aligned}$$

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3. Compute the following integrals

(a) (10 points)

$$\int_0^{\pi} \tan\left(\frac{x}{3}\right)\, dx$$

(b) (10 points)

$$\int_2^4 (1+\ln t)t \ln t\, dt$$

(c) (10 points)

$$\int \tan x \ln(\cos x)\, dx$$

Solution:

(a) $u = x/3$ and then $v = \cos u$.

$$\int_0^{\pi} \tan(x/3)\, dx = 3\int_0^{\pi/3} \frac{\sin u}{\cos u}\, du = -3\ln|\cos u|\Big|_0^{\pi/3} = -3\ln(1/2) = 3\ln 2.$$

(b) $u = t\ln t$; $du = (\ln t + 1)dt$.

$$\int_2^4 (1+\ln t)t\ln t\, dt == \int_{2\ln 2}^{4\ln 4} u\, du == \frac{1}{2}[16(\ln 4)^2 - 4(\ln 2)^2] = 8(\ln 4)^2 - 2(\ln 2)^2 = 30(\ln 2)^2.$$

(c) $u = \ln(\cos x)$; $du = -\tan x\, dx$.

$$\int \tan x \ln(\cos x)\, dx = -\int u\, du = -\frac{1}{2}[\ln(\cos x)]^2 + C.$$

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4. (25 points) (7.2.30) If

$$y' = -\alpha y, \quad y(30) = 10^6, \quad y(40) = 10^3,$$

then find $y(0)$.

Solution: $y(t) = ce^{-\alpha t}$. Therefore,

$$ce^{-30\alpha} = 10^6$$
$$ce^{-40\alpha} = 10^3$$

and

$$e^{10\alpha} = 10^3.$$

It follows that

$$c(10^3)^{-3} = 10^6$$

or

$$c = y(0) = 10^{15}.$$