Math 1552 Material to Cover on Monday

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1 Administration

Please point the students to the course webpage

http://www.math.gatech.edu/ mccuan/courses/1552/

and hand out copies of the "syllabus," which is just a printout of that course page.

In addition, you can tell them about the MyMethLab approach to doing homework. They'll need to know how to login, and there is a handout which can be printed also from the course webpage. Once they find TheirMethLab, the important piece of information is:

Course ID: mccuan54436

2 Other Stuff

You can also do a review of differentiation along the following lines:

Ask them to find the derivatives of the following functions. (You can ask them to write down the answers on paper, for example, and see if they can do it faster than you can write them on the board. Some of them can do with an explanation as well, and they might have questions.)

1. $f(x) = 3x^5$

   Ans. $f'(x) = 15x^4$

2. $f(x) = 7$

   Ans. $f'(x) = 0$.

3. $f(x) = 2^x$

   Ans. $f'(x) = 2^x \ln 2$ (Note: $2^x = e^{\ln 2^x} = e^{x \ln 2}$, so the factor of $\ln 2$ comes from the chain rule.)

4. $f(x) = 1/x$

   Ans. $f'(x) = -1/x^2$

5. $f(x) = \sin x$

   Ans. $f'(x) = \cos x$

6. $f(x) = \sec x$

   Ans. $f'(x) = \sec x \tan x$

7. $f(x) = \ln x$

   Ans. $f'(x) = 1/x$

8. $f(x) = (\ln x)^2$

   Ans. $f'(x) = 2 \ln x/x$

9. $f(x) = \tan^{-1}(x)$

   Ans. $f'(x) = 1/(1+x^2)$. (Note: $\tan(\tan^{-1}(x) = x$, so by the chain rule

   $$\sec^2(\tan^{-1}(x))\frac{d}{dx}\tan^{-1}(x) = 1.$$

   On the other hand, you can draw a right triangle to convince yourself that if $\theta = \tan^{-1}(x)$, i.e., if θ is an angle with tangent x (say opposite side x and adjacent side 1), then $\sec\theta = \sqrt{1+x^2}$. In particular,

   $$\sec^2(\tan^{-1}(x)) = 1 + x^2.)$$

Having gone through those, you can point out that any time you know how to differentiate a function, you also know how to antidifferentiate (or integrate) the derivative. So you can ask them the following:

Find a function whose derivative is the given function:

1. $f'(x) = x^2$

   Ans. $f(x) = x^3/3$

2. $f'(x) = 0$

   Ans. At this point, we might notice that the answer could be any constant, so $f(x) = 1$ works or $f(x) = C$ for any constant C.

   Are there any other possibilities? Answer: No.

   So we might make the question a little "harder" by asking:

   Find *all possible* functions whose derivative is the given function.

3. $f'(x) = e^{2x}$

   Ans. $f(x) = e^{2x}/2 + C$ (Note: You have to anticipate the use of the chain rule here—or something.)

4. $f'(x) = 1/x$

   Ans. $f(x) = \ln x + C$ (Well, that's fine if $x > 0$. One can check that you need $f(x) = \ln(-x)$ if $x < 0$.)

5. $f'(x) = \sin x$

   Ans. $f(x) = -\cos x + C$

6. $f'(x) = \sec^2 x$

   Ans. $f(x) = \tan x + C$

7. $f'(x) = 3/x^2$

   Ans. $f(x) = -3/x + C$

8. $f'(x) = (\ln x)/x$

   Ans. $f(x) = (\ln x)^2/2 + C$ (Note: We know from our differentiation, if we remember, that the derivative of $(\ln x)^2$ is $2\ln x/x$ which is just what we want—up to a factor of 2.)

9. $f'(x) = 1/\sqrt{1-x^2}$

   Ans. $f(x) = \sin^{-1}(x) + C$. (Note: This one "most people" just remember.)

As a final exercise (and another example of one that is just supposed to be remembered) you can have them calculate the derivative of the function

$$f(x) = \ln(\sec x + \tan x).$$

Of course, $f'(x) = \sec x$. Of course this means something like

$$\int \sec x \, dx = \ln(\sec x + \tan x) + C$$

though that is not quite correct because $\ln(\sec x + \tan x)$ is not well-defined everywhere.

Then you can ask where f is well-defined. The answer turns out to be

$$\cup_{j=-\infty}^{\infty}((2j - 1/2)\pi, (2j + 1/2)\pi).$$

Drawing some nice pictures at this point can be helpful. Also, if you want to get into details, the fact that

$$\sec x + \tan x = \frac{1 + \sin x}{\cos x}$$

(and the numerator is mostly positive) can be helpful.