**Math 1552, Exam 2: Integration (practice)** Name and section: ____________________

1. (20 points) (6.1.53) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about the line $x = 1$. Hint: Use the method of washers.

**Solution:** The outer radius of the washer is $1+\sqrt{z}$ while the inner radius is $1-\sqrt{z}$. Thus, the volume is

$$\begin{aligned} V &= \int_0^1 \left[\pi(1+\sqrt{z})^2 - \pi(1-\sqrt{z})^2\right]\, dz \\ &= 4\pi \int_0^1 \sqrt{z}\, dz \\ &= \frac{8\pi}{3} \end{aligned}$$

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2. (20 points) (6.2.25) Use the method of cylindrical shells to find the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about the $y$-axis.

**Solution:** The shells are indexed by $x \in [0, 1]$, and the height of the shell at radius $x$ is $x - x^2$. Therefore, the volume by cylindrical shells is

$$\begin{aligned} V &= \int_0^1 2\pi x(x - x^2)\, dx \\ &= 2\pi \int_0^1 (x^2 - x^3)\, dx \\ &= 2\pi(1/3 - 1/4) \\ &= \frac{\pi}{6}. \end{aligned}$$

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3. (20 points) (6.3.20) Find the length of the graph of $y = \ln(\cos x)$ between $x = 0$ and $x = \pi/4$.

**Solution:** We can parameterize this curve by $x(t) = t$ and $y(t) = \ln(\cos t)$ for $0 \leq t \leq \pi/4$. The speed of the parameterization is

$$\sqrt{x'(t)^2 + y'(t)^2} = \sqrt{1 + \tan^2 t} = \sec t.$$

Thus the length is

$$\begin{aligned} L &= \int_0^{\pi/4} \sec t \, dt \\ &= \ln(\sec t + \tan t)\Big|_0^{\pi/4} \\ &= \ln(\sqrt{2} + 1) - \ln(1) \\ &= \ln(1 + \sqrt{2}). \end{aligned}$$

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4. (7.1.4,7,19,30) Compute the following integrals

   (a) (5 points)

$$\int \frac{8t}{4t^2 - 5}\, dt$$

   (b) (5 points)

$$\int \frac{1}{2\sqrt{x} + 2x}\, dx$$

   (c) (5 points)

$$\int \frac{e^{1/t}}{t^2}\, dt$$

   (d) (5 points)

$$\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$$

**Solution:**

(a) $u = 4t^2 - 5$.

$$\int \frac{8t}{4t^2 - 5}\, dt = \int \frac{1}{u}\, du = \ln|u| + C = \ln|4t^2 - 5| + C.$$

(b) $u = 1 + \sqrt{x}$

$$\int \frac{1}{2\sqrt{x} + 2x}\, dx = \int \frac{1}{2} \frac{1}{\sqrt{x}(1 + \sqrt{x})}\, dx = \int \frac{1}{u}\, du = \ln(1 + \sqrt{x}) + C.$$

(c) $u = 1/t$.

$$\int \frac{e^{1/t}}{t^2}\, dt = -\int e^u\, du = -e^{1/t} + C.$$

(d) $u = \sqrt{x}$.

$$\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx = \int_1^2 2 \cdot 2^u\, dx = 2^{u+1}/\ln 2\Big|_1^2 = (8 - 4)/\ln 2 = 4/\ln 2.$$

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5. (20 points) (7.2.3) Solve the initial value problem

$$y' = -0.6y, \quad y(0) = 100.$$

**Solution:** $y(t) = ce^{-0.6t}$ and $y(0) = 100$. Therefore,

$$y(t) = 100e^{-0.6t}.$$