Math 2403, Final Exam (practice) Name and section: _____

1. (a) (5 points) Solve the IVP

$$\begin{cases} y' = y^2 \\ y(2) = 1. \end{cases}$$

(b) (5 points) Plot the solution you found in part (a).

(c) (5 points) What is the *interval of definition* of your solution you found in part (a)?

(d) (5 points) Plot the phase line for the ODE $y' = y^2$.

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2. A series circuit contains a 2 Ohm resistor, a 1/48 Fahrad capacitor, a 0.02 Henry inductor, and an adjustable power source.

   (a) (10 points) If the initial charge on the capacitor is 1/16 Coulomb and there is initially no current flowing in the circuit when the power source is switched on to 9 volts, what is the subsequent charge on the capacitor?

   (b) (10 points) Does this physical system constitute an oscillator? Explain.

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3. Consider the system of ODEs

$$y' = y(5 - y + z)$$
$$z' = -z(5 - z + y)^2.$$

(a) (10 points) Linearize at $y_* = z_* = 0$, and draw the phase diagram for the *linerized* system.

(b) (10 points) A solution of the original nonlinear system starts with $y(0) = 0.1$ and $z(0) = 0$. Determine the limit

$$\lim_{t \nearrow \infty} y(t).$$

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4. (20 points) Solve the initial value problem

$$\begin{cases} y'' + 4y = \cos 3t \\ y(0) = 0 = y'(0) \end{cases}$$

determine the *period* of the beats.

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5. (20 points) Solve the initial value problem

$$\begin{cases} y'' + 2y' - y = t \\ y(0) = 0 = y'(0) \end{cases}$$

by the method of Laplace transforms.