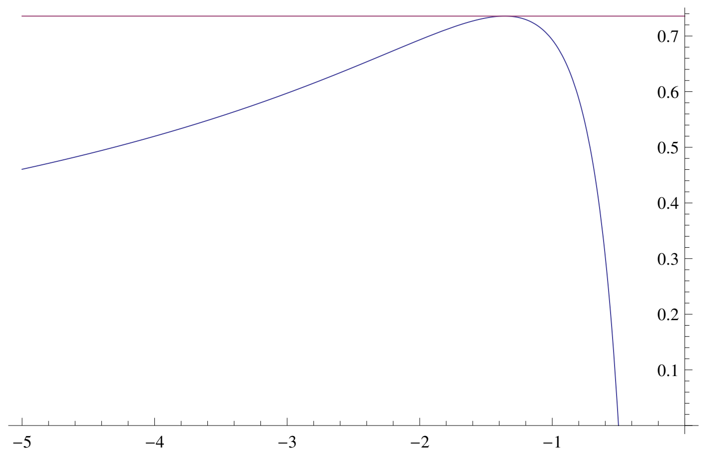
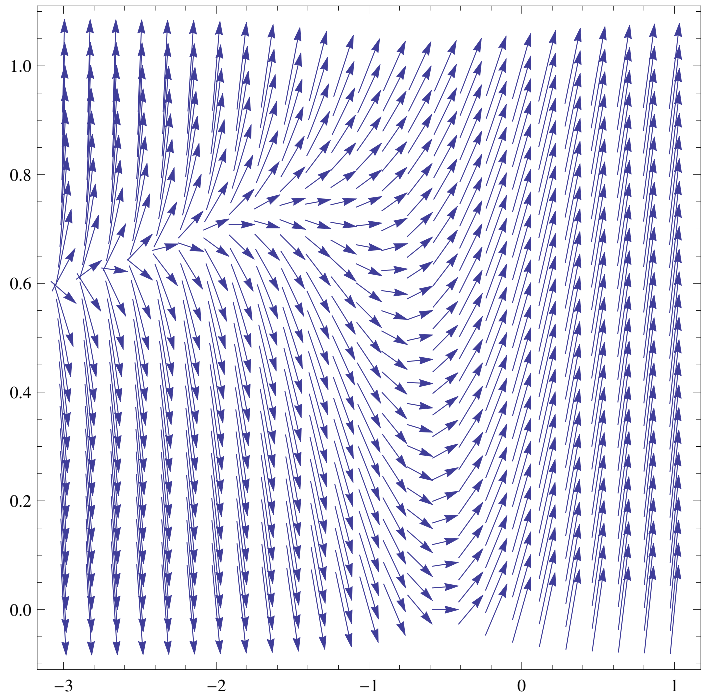
A Homework Problem (continued)

Here, I want to explore the IVP in problem 2.6.4 of Brannon and Boyce numerically. Let's start by illustrating some of the things we've already determined without numerics.

```
g[x_] = Log[-1 / (2 x)] / x;
a = Plot[{g[x], 2 / E}, {x, -5, 0}, AxesOrigin -> {0, 0}]
```

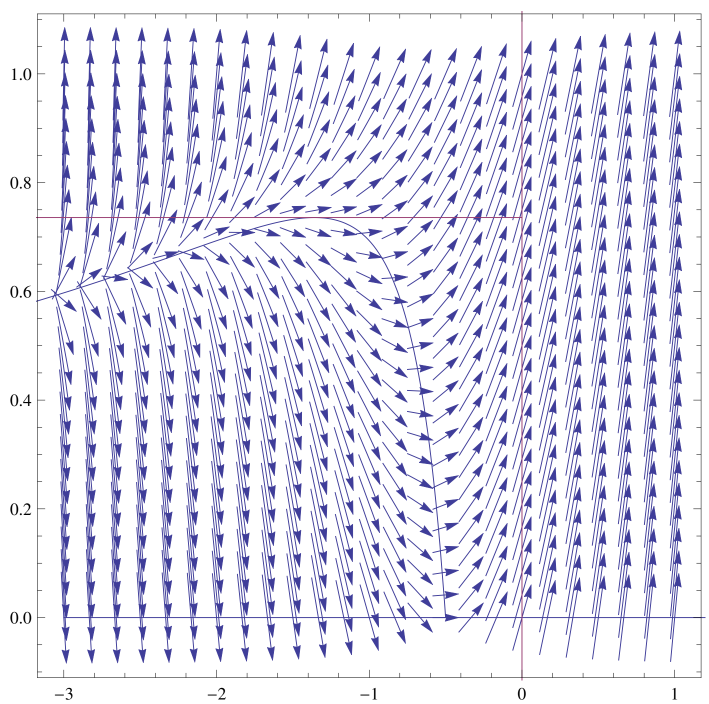


```
b = VectorPlot[{1, 2 t + E^(-y t)} / (10 Norm[{1, 2 t + E^(-y t)}]),
  {t, -3, 1}, {y, 0, 1}, VectorScale → Automatic, VectorPoints → Fine]
```




```
c = ParametricPlot[{{t, 0}, {0, t}}, {t, -3, 3}];
```

```
Show[b, a, c]
```

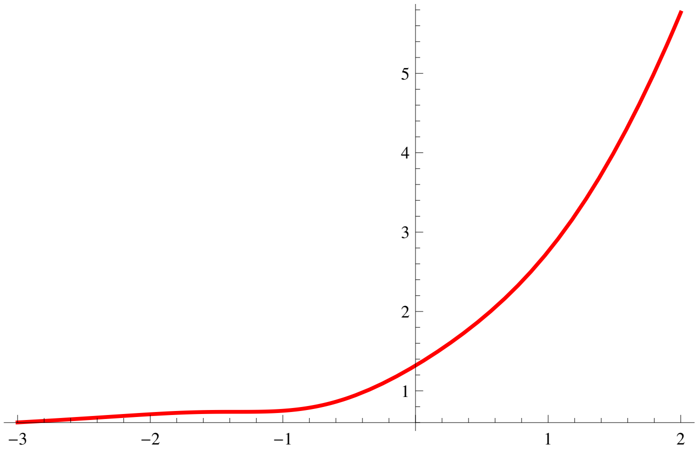


This seems to indicate what is going on with the slope field pretty well. Next, we find some solutions. First the critical solution which satisfies initial condition y(-e/2) = 2/e. It ought to go through the maximum of the graph of g and always stay above that graph otherwise:

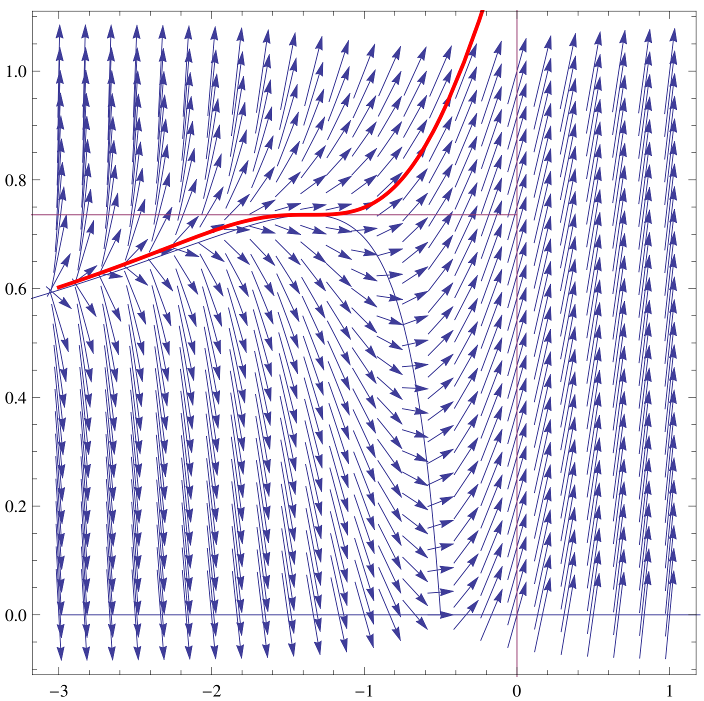
```
csoln = NDSolve[{y'[t] == 2 t + E^(-t y[t]), y[-E / 2] == 2 / E}, y, {t, -4, 2}]
```

```
{{y → InterpolatingFunction[{{-4., 2.}}, <>]}}
```

```
d = Plot[y[t] /. csoln, {t, -3, 2}, PlotStyle → {Thick, Red}]
```




```
Show[b, a, c, d]
```

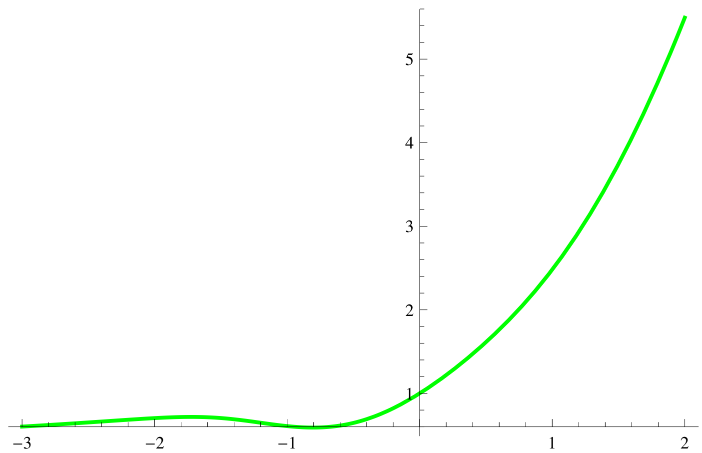


From this, it' s clear that the solution of our IVP will fall under the critial solution at t = 0 and for all time. It will be defined (presumably) for all time. It will first increase from zero (maybe for t really negative), then cross the graph of g (this may be hard to see), decrease for a while, then cross the graph of g again and increase.

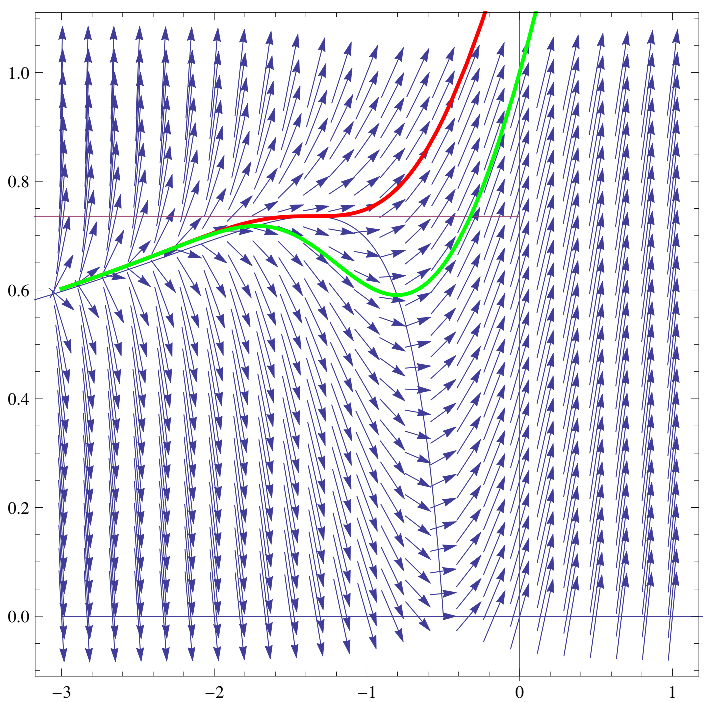
```
soln = NDSolve[{y'[t] == 2 t + E^(-t y[t]), y[0] == 1}, y, {t, -4, 2}]
```

```
{{y → InterpolatingFunction[{{-4., 2.}}, <>]}}
```

```
e = Plot[y[t] /. soln, {t, -3, 2}, PlotStyle → {Thick, Green}]
```

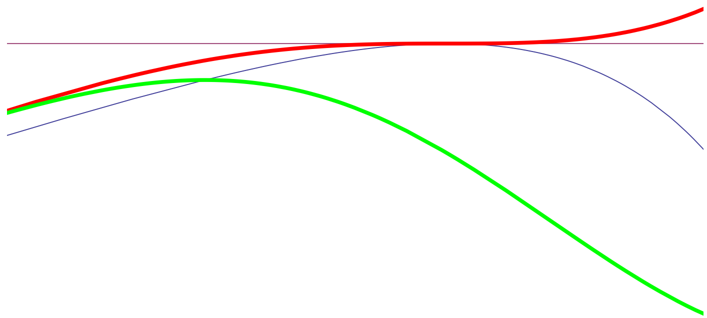



```
Show[b, a, c, d, e]
```



Well, there it is, and it looks like it crosses the graph of g within our window, so let's see if we can zoom in and see the intersection point, which will be a local max for the solution of the IVP.

```
Show[a, d, e, PlotRange → {{-2, -1}, {.6, .8}}]
```



I guess that captures it pretty well. The green curve will always stay below the red curve (by the existence and uniqueness theorem).