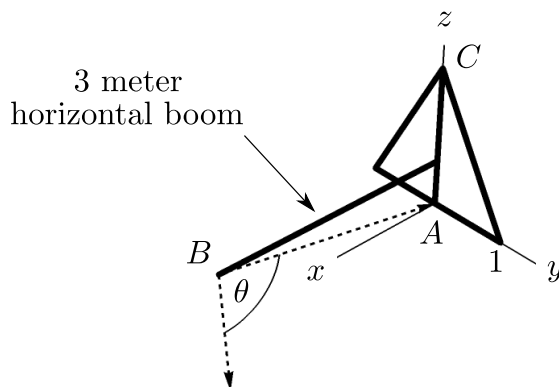


1. (12.3.6) The boom on a crane is constructed of steel pipes drawn with the thick lines in the schematic drawing below. The triangular support is an equilateral triangle of side length 2 meters in the  $y, z$ -plane as indicated in the drawing. The boom is 3 meters long and extends horizontally orthogonal to the  $y, z$ -plane from the centroid of the triangle. The cable passes through points  $A = (0, 0, 0)$  and  $B$  and hangs down vertically from  $B$  as indicated by the dashed vectors. Note: The **centroid** of an equilateral triangle is the intersection of the medians, the intersection of the angle bisectors, and the intersection of the altitudes. (These are all the same point for an equilateral triangle.) A **median** of a triangle is the line segment joining a vertex with the midpoint of the opposite side.



- (a) (7 points) Find the distance from  $B$  to  $C$  in meters.
- (b) (8 points) Find the cosine of the angle  $\theta$  between the two straight portions of the cable.
- (c) (10 points) Let  $\mathbf{v}$  be the vector displacement from  $C$  to  $B$ , and let  $\mathbf{w}$  be the vector displacement from  $C$  to  $(0, 1, 0)$ . Find  $\mathbf{v} \times \mathbf{w}$ .

Name and section: \_\_\_\_\_

2. (quadric surfaces) Sketch the quadric surfaces in  $\mathbb{R}^3$  associated with the relations:

(a) (15 points)  $x^2 - 16y^2 - 4z^2 = 64$

(b) (10 points)  $z = -2x^2$

Name and section: \_\_\_\_\_

3. (25 points) (13.2.16) Solve the initial value problem for the vector valued function  $\mathbf{r}$ .

$$\begin{cases} \mathbf{r}' = (t/(t^2 + 2), (t + 1)/(2 - t), (t^2 + 4)/(t^2 + 3)) \\ \mathbf{r}(0) = (3, -2, 1). \end{cases}$$

Remember the indefinite integral  $\int 1/(\tau^2 + 1) d\tau = \tan^{-1} \tau + C$ .

Name and section: \_\_\_\_\_

4. (motion along a helix) A point mass moves along a cylinder with position given as a function of time by

$$\mathbf{r}(t) = (5 \cos \theta, -5 \sin \theta, t/2)$$

where  $\theta = \theta(t)$ .

- (a) (10 points) Find the velocity and acceleration on the mass.

- (b) (10 points) Assuming  $\theta(t) = t$ , draw a picture of the path of the point mass.

Name and section: \_\_\_\_\_

5. (Bonus 10 points) (curvature) Find the curvature of the path parameterized by

$$\mathbf{r}(t) = (5 \cos t, -5 \sin t, t/2).$$