Lecture 9 (F sections)

EXAM

– Numerics

+ Systems
  - Existence/uniqueness
  - Linear / nonlinear
  - solve constant coeff. linear
    - basis of eigenvectors.

1.

3.3.1

$$\mathbb{X}' = \underbrace{\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}}_{A} \mathbb{X} = \mathbb{F}(\mathbb{X})$$

$$\det(A - \lambda I) = (3-\lambda)(-2-\lambda) + 4$$

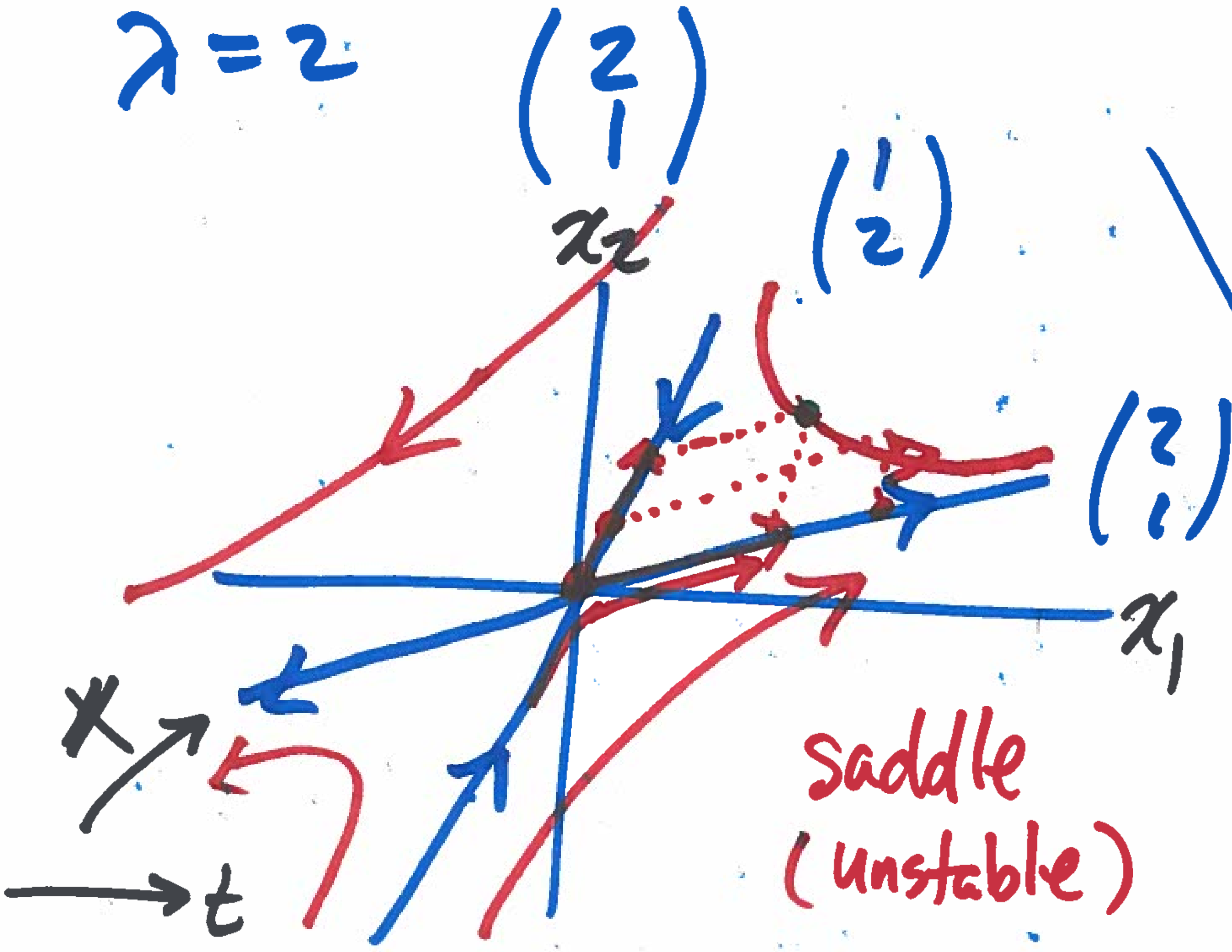
$$= \lambda^2 - \lambda - 2 = (\lambda - 2)(\lambda + 1)$$

$\lambda = -1$ or 2.

$\lambda = -1$

$$\begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \boxed{v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

$\lambda = 2$



$$\mathbb{F}\begin{pmatrix} x_1^* \\ x_2^* \end{pmatrix} = \begin{pmatrix} 3x_1^* - 2x_2^* \\ 2x_1^* - 2x_2^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A\mathbb{X}_* = \mathbb{O}$$

$$\mathbb{X}(t) = c e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbb{X}(t) = c e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\mathbb{X}(t) = c_1 e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Orbit **Trajectory** (Dsfn)

$\mathbb{X}' = \mathbb{F}(\mathbb{X})$, $\mathbb{X} = \mathbb{X}(t)$

particular solution.

$$\underbrace{\{\mathbb{X}(t) : t \in \mathbb{R}\}}_{\text{orbit or trajectory}} \subseteq \text{phase space}$$

4.

<u>3.3.25</u>

$$\underbrace{y'' + 7y' + 10y}_{Ly} = 0$$

$x_1 = y$

$x_2 = y'$

$$\begin{cases} x_1' = x_2 \\ x_2' = -10x_1 - 7x_2 \end{cases}$$

$$\mathbb{x}' = \begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix} \mathbb{x}$$

$$\lambda^2 + 7\lambda + 10 = (\lambda + 2)(\lambda + 5)$$

5.

$y'' + 7y' + 10y = 0$

$\uparrow$

$y(t) = c_1 e^{-5t} + c_2 e^{-2t}$.

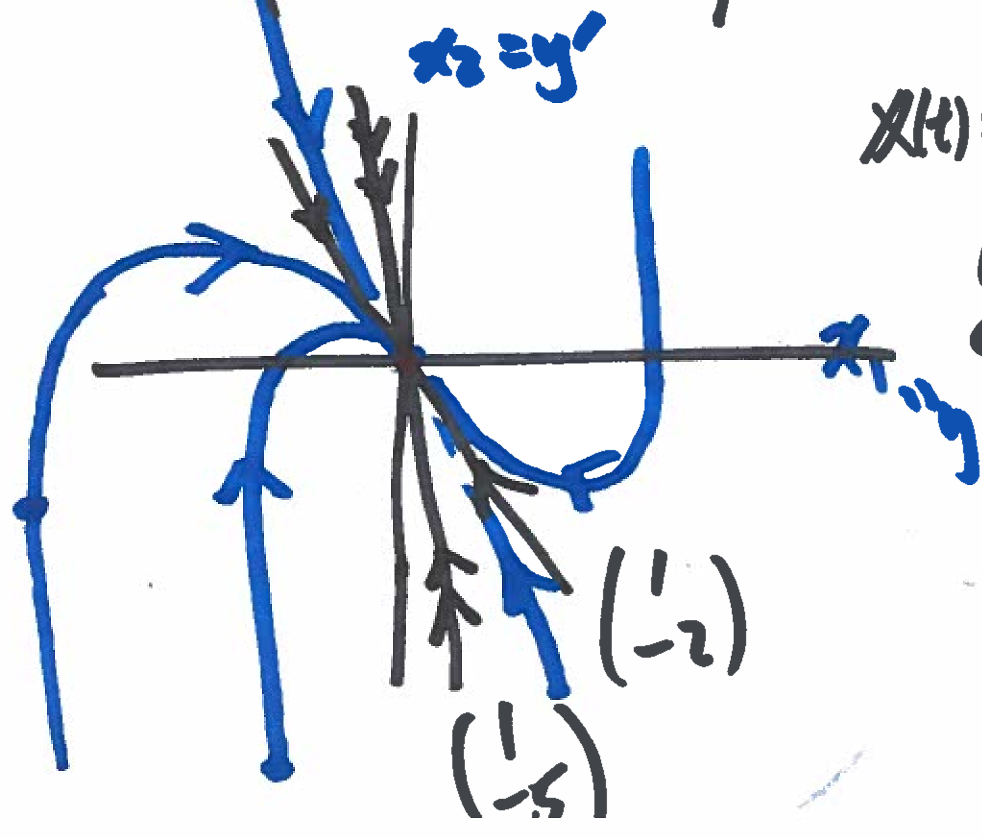
$\vec{x}(t) = c_1 e^{-5t}\begin{pmatrix}1\\-5\end{pmatrix} + c_2 e^{-2t}\begin{pmatrix}1\\-2\end{pmatrix}$

$\underline{\lambda = -2}$

$$\begin{pmatrix}2 & 1\\-10 & -5\end{pmatrix} \qquad \begin{pmatrix}1\\-2\end{pmatrix}$$

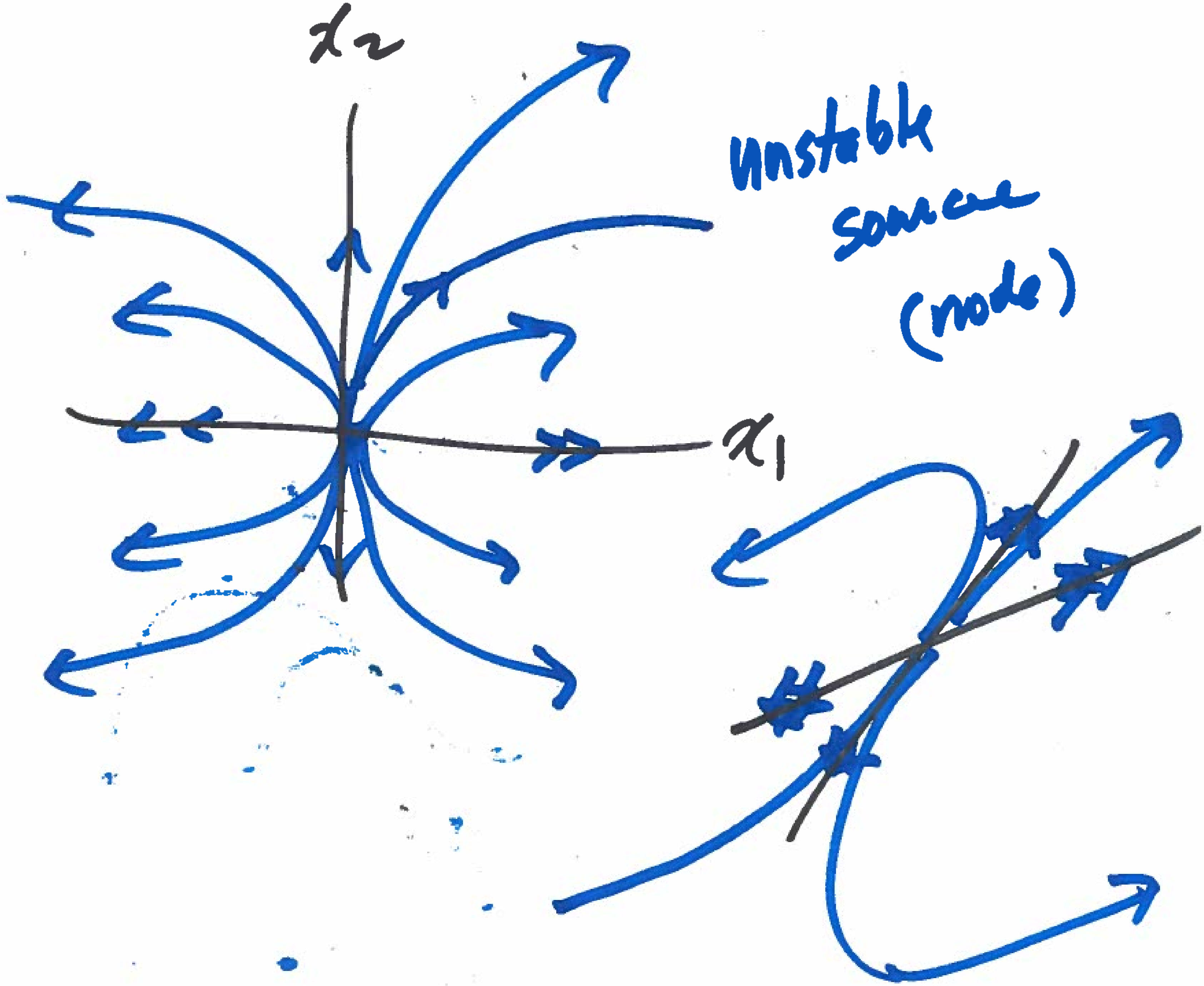
$\underline{\lambda = -5}$

$$\begin{pmatrix}5 & 1\\-10 & -2\end{pmatrix} \qquad \begin{pmatrix}1\\-5\end{pmatrix}$$



stable
"sink"

6.



unstable source (node)