Lecture 9 (L)

− numerical methods

+ systems

- Existence/uniqueness
- Linear
- phase diagrams, direction field.
- Linear constant coefficient
  - basis eigenvectors

3.3.25

$$\underbrace{y'' + 7y' + 10y}_{\text{linear } Ly} = 0$$

$$\begin{cases} x_1 = y \\ x_2 = y' \end{cases}$$

$$\begin{cases} x_1' = x_2 \\ x_2' = -10x_1 - 7x_2 \end{cases}$$

$$\mathbb{X}' = \begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix} \mathbb{X}$$

$$\mathbb{X}' = \mathbb{F}(\mathbb{X})$$

$$\underline{\underline{\mathbb{F}(\mathbb{X}_*) = \mathbb{O}}}$$

$$\mathbb{X}' \doteq \begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix} \mathbb{X} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\mathbb{X}_* = -\begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix}^{-1} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\mathbb{F}(\mathbb{X}_*) = \begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix} \mathbb{X}_* \doteq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbb{X}_* \doteq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\lambda^2 + 7\lambda + 10 = (\lambda+5)(\lambda+2) = 0$$

$\underline{\lambda = -5}$ $\quad \begin{pmatrix} 5 & 1 \\ -10 & -2 \end{pmatrix} \quad \mathbb{V} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

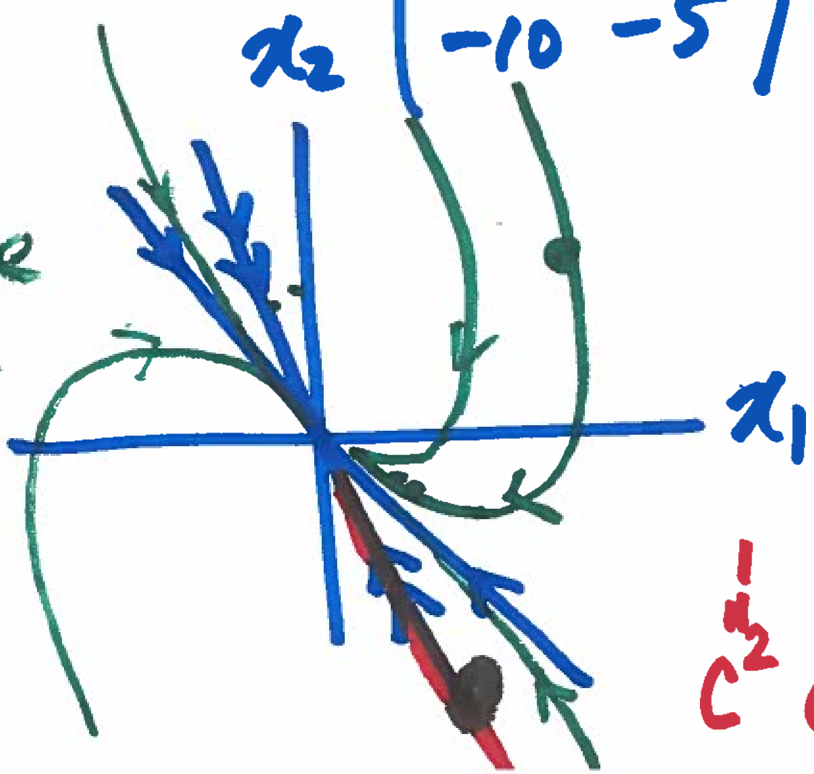
$\underline{\lambda = -2}$ $\quad \begin{pmatrix} 2 & 1 \\ -10 & -5 \end{pmatrix} \quad \mathbb{W} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\boxed{\mathbb{X}(t) = c_1 e^{-5t}\begin{pmatrix} 1 \\ -5 \end{pmatrix} + c_2 e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}}$$

$$y(t) = c_1 e^{-5t} + c_2 e^{-2t}$$

$$y'' + 7y' + 10y = 0$$

$$y' = -5c_1 e^{-5t} - 2c_2 e^{-2t} \checkmark$$



Stable Sink

$c_1 e^{-5t}\begin{pmatrix} 1 \\ -5 \end{pmatrix}$

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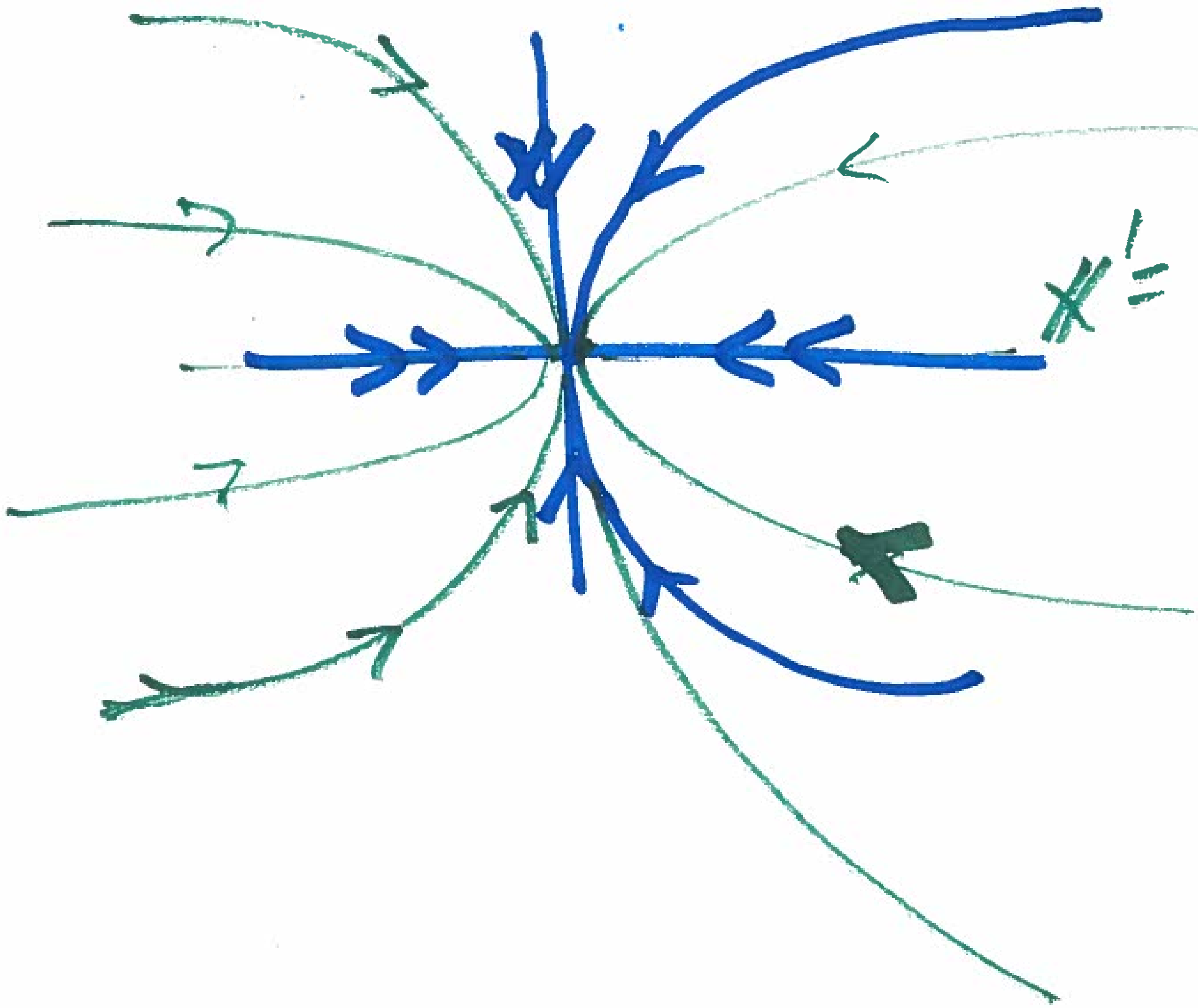
$$\mathbf{x}' = \begin{pmatrix} -5 & 0 \\ 0 & -2 \end{pmatrix} \mathbf{x}$$



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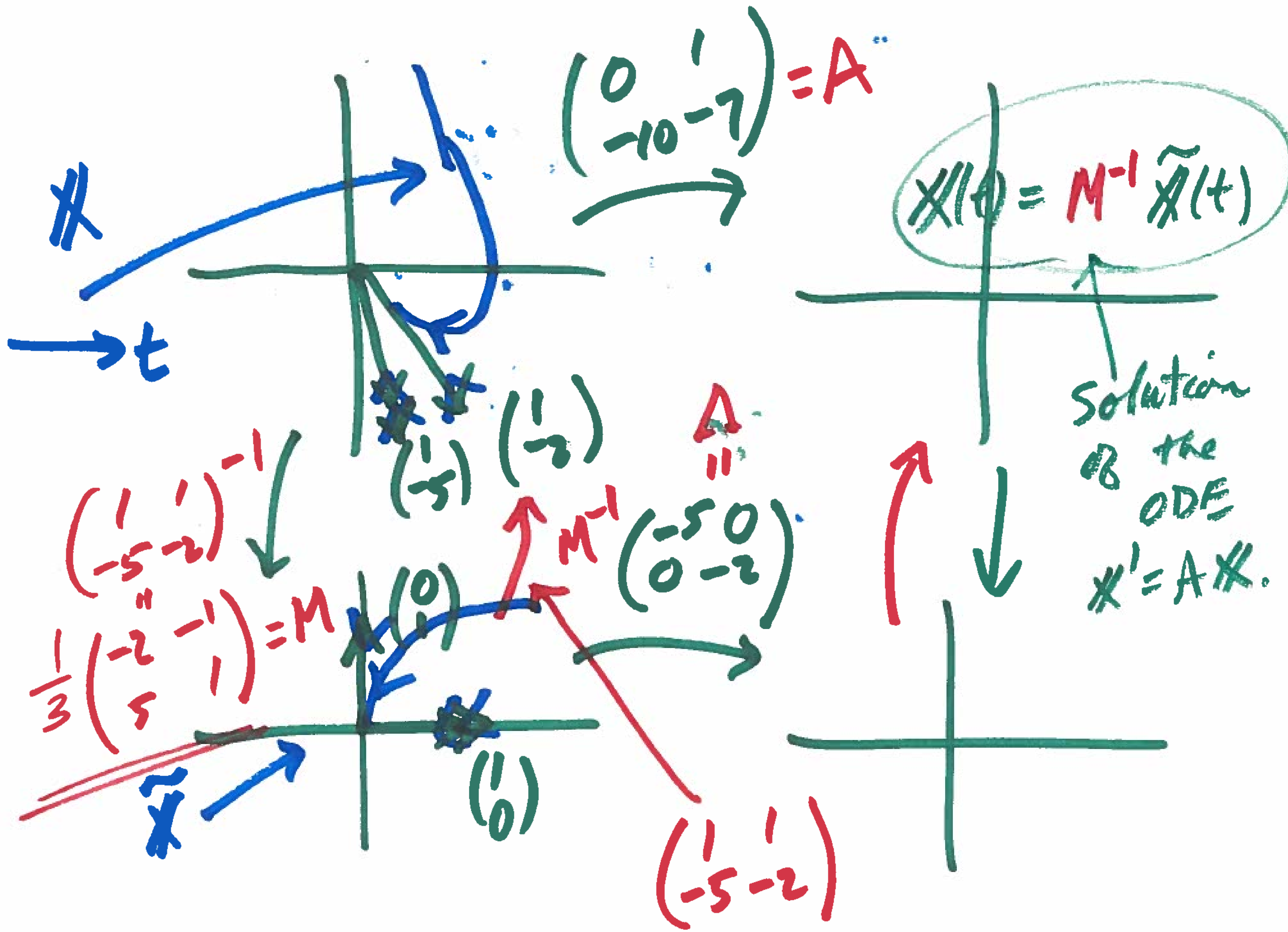


$\mathbb{X}$
$\to t$
$\begin{pmatrix} 0 & 1 \\ -10 & -7 \end{pmatrix} = A$
$\mathbb{X}(t) = M^{-1}\tilde{\mathbb{X}}(t)$
Solution of the ODE $\mathbb{X}' = A\mathbb{X}$.
$\begin{pmatrix} 1 \\ -5 \end{pmatrix}$
$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
$A = M^{-1}\begin{pmatrix} -5 & 0 \\ 0 & -2 \end{pmatrix}$
$\begin{pmatrix} 1 & 1 \\ -5 & -2 \end{pmatrix}^{-1} = \frac{1}{3}\begin{pmatrix} -2 & -1 \\ 5 & 1 \end{pmatrix} = M$
$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\tilde{\mathbb{X}}$
$\begin{pmatrix} 1 & 1 \\ -5 & -2 \end{pmatrix}$

b

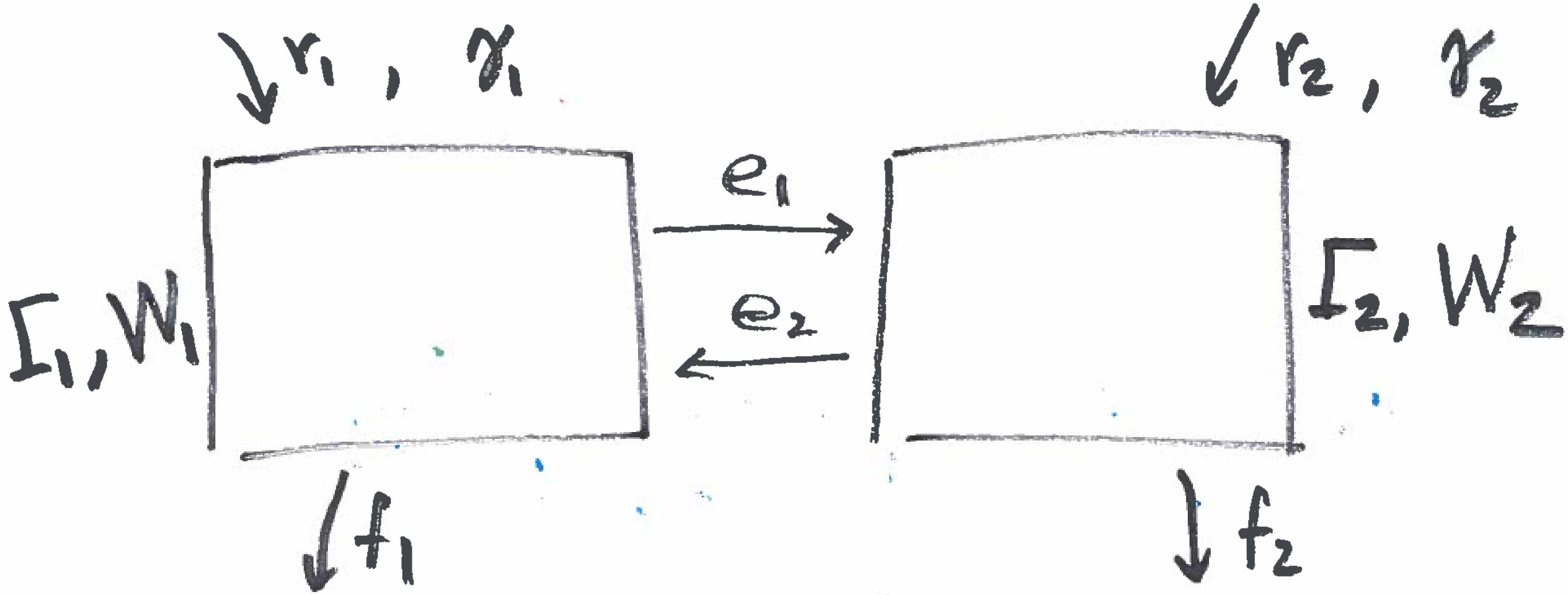
$$MAM^{-1} = \Lambda$$

$$\tilde{\mathbb{x}}' = \Lambda \tilde{\mathbb{x}} = \begin{pmatrix} -5 & 0 \\ 0 & -2 \end{pmatrix} \tilde{\mathbb{x}}$$

$$\tilde{x}_1(t) = c_1 e^{-5t}$$

$$\tilde{x}_2(t) = c_2 e^{-2t}$$

$$\tilde{\mathbb{x}} = c_1 e^{-5t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



① IVP

$$S_1' = r_1\gamma_1 - f_1 \frac{S_1}{V_1} - e_1 \frac{S_1}{V_1} + e_2 \frac{S_2}{V_2}; \quad S_1(0) = \Gamma_1 W_1$$

$$V_1 = W_1 + r_1 t - e_1 t + e_2 t - f_1 t$$

$$V_2 = W_2 + r_2 t + e_1 t - e_2 t - f_2 t$$

$$S_2' = r_2\gamma_2 + e_1 \frac{S_1}{V_1} - e_2 \frac{S_2}{V_2} - f_2 \frac{S_2}{V_2}; \quad S_2(0) = \Gamma_2 W_2$$

$$\mathbb{x} = M^{-1}\tilde{\mathbb{x}}$$

$$\tilde{\mathbb{x}}' = \Lambda\tilde{\mathbb{x}}$$

$$\uparrow$$

$$= MAM^{-1}\tilde{\mathbb{x}}$$

$$\mathbb{x}' = M^{-1}\tilde{\mathbb{x}}'$$

$$= M^{-1}(MAM^{-1})\tilde{\mathbb{x}}$$

$$= A(M^{-1}\tilde{\mathbb{x}})$$

$$= A\mathbb{x}$$