Final Exam 2015

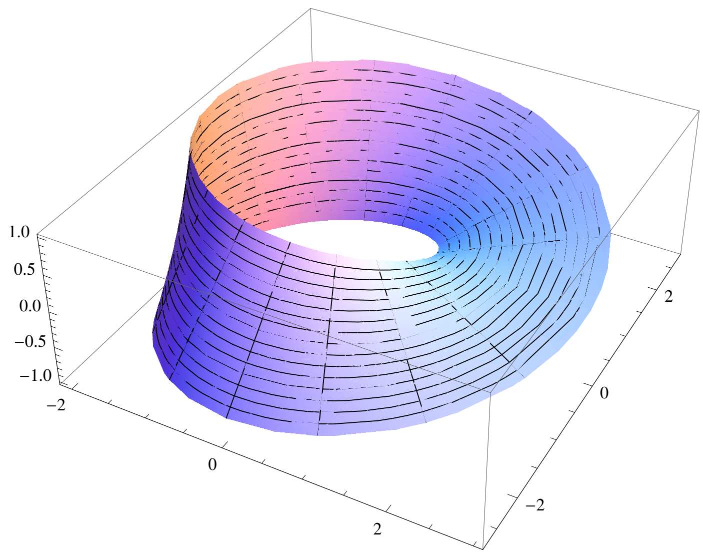
2.

In[1]:=
```
X2[u_, v_] = {(2 + v Cos[u / 2]) Cos[u], (2 + v Cos[u / 2]) Sin[u], v Sin[u / 2]}
```

Out[1]= $\left\{\left(2+v\,\mathrm{Cos}\left[\frac{u}{2}\right]\right)\mathrm{Cos}[u],\ \left(2+v\,\mathrm{Cos}\left[\frac{u}{2}\right]\right)\mathrm{Sin}[u],\ v\,\mathrm{Sin}\left[\frac{u}{2}\right]\right\}$

In[16]:=
```
ParametricPlot3D[X2[u, v], {u, 0, 2 Pi}, {v, -1, 1}]
```

Out[16]=



In[3]:=
```
X2u[u_, v_] = D[X2[u, v], u]
X2v[u_, v_] = D[X2[u, v], v]
```

Out[3]= $\left\{-\frac{1}{2}v\,\mathrm{Cos}[u]\,\mathrm{Sin}\left[\frac{u}{2}\right]-\left(2+v\,\mathrm{Cos}\left[\frac{u}{2}\right]\right)\mathrm{Sin}[u],\ \left(2+v\,\mathrm{Cos}\left[\frac{u}{2}\right]\right)\mathrm{Cos}[u]-\frac{1}{2}v\,\mathrm{Sin}\left[\frac{u}{2}\right]\mathrm{Sin}[u],\ \frac{1}{2}v\,\mathrm{Cos}\left[\frac{u}{2}\right]\right\}$

Out[4]= $\left\{\mathrm{Cos}\left[\frac{u}{2}\right]\mathrm{Cos}[u],\ \mathrm{Cos}\left[\frac{u}{2}\right]\mathrm{Sin}[u],\ \mathrm{Sin}\left[\frac{u}{2}\right]\right\}$

```
In[5]:= g11[u_, v_] = Simplify[X2u[u, v].X2u[u, v]]
        F = Simplify[X2u[u, v].X2v[u, v]]
        G = Simplify[X2v[u, v].X2v[u, v]]
```

Out[5]= $4 + \frac{3\,v^2}{4} + 4\,v\,\mathrm{Cos}\left[\frac{u}{2}\right] + \frac{1}{2}\,v^2\,\mathrm{Cos}[u]$

Out[6]= 0

Out[7]= 1

```
Integrate[Sqrt[g11[u, v]], {u, 0, 2 Pi}]
```

$$\int_0^{2\pi} \sqrt{4 + \frac{3\,v^2}{4} + 4\,v\,\mathrm{Cos}\left[\frac{u}{2}\right] + \frac{1}{2}\,v^2\,\mathrm{Cos}[u]}\;du$$

```
Integrate[Sqrt[g11[u, v]], {v, -1, 1}]
```

```
In[8]:= halfarea = NIntegrate[Sqrt[g11[u, v]], {u, 0, 2 Pi}, {v, -1, 1}]
```

Out[8]= 25.4131

```
In[14]:= halfarea (36 / 125)
```

Out[14]= 7.31897

This means you need at least 8 quarts or 2 gallons.

competitors

A first possible “easy” but wrong approach would be to assume the area is the same as a circular cylinder of radius 2 and height 2:

```
cylinderarea = N[2 Pi (2) (2)]
```

Out[12]= 25.1327

The total paint in this case would be

```
In[13]:= cylinderarea (36 / 125)
```

Out[13]= 7.23823

So you would still buy two gallons, so it turns out this is an adequate approximation for this application. Nevertheless, the calculated area is a little less than the actual area---but only by about half a square foot.

A second possibility would be to again assume you have a metric rectangle of width 2 but length the length of half the boundary curve.

```
In[10]:= boundarylength = NIntegrate[Sqrt[g11[u, 1]], {u, 0, 2 Pi}]
```

Out[10]= 13.0068

```
In[11]:= 2 boundarylength
```

Out[11]= 26.0135

In[15]:=
```
2 boundarylength (36 / 125)
```

Out[15]=
```
7.49189
```

This gives an overestimate, though for practical purposes it is still adequate. You’re still going to buy two gallons of paint.