Math 4441, Final Exam: Differential Geometry (practice) Name/Section: ____________________

1. (25 points) (1-5.12) Let $\gamma : [0,1] \to \mathbb{R}^3$ be a regular parameterized curve. Show that the curvature at $\gamma = \gamma(t)$ is

$$k = k(t) = \frac{|\gamma' \times \gamma''|}{|\gamma'|^2}.$$

Solution:

2. (25 points) (1-5.11; alternative for first problem) A planar curve is parameterized by

$$\gamma(\theta) = (r(\theta)\cos\theta, r(\theta)\sin\theta).$$

Find an expression for the curvature of γ in terms of the given function r. (Simplify your answer completely.)

Alternative: If the curvature of γ is written as

$$k(\theta) = \frac{ar'^2 + brr'' + r^2}{(r^2 + r'^2)^{3/2}}$$

where a and b are constants, find the values of a and b.

Solution:

3. (25 points) (1-5.4) If all (principal) normals of a parameterized planar curve pass through a given point, show that the curve is contained in a circle.

Solution:

4. (25 points) (2-5.10) Compute the first fundamental form of polar coordinates on the plane.

Solution:

5. (25 points) (2-2.1-3, 2-3.1-3, 3-3.15) Find a surface with constant mean curvature 3 which is diffeomorphic to

$$\mathcal{S} = \{(x, y, 0) : 1 < x^2 + y^2 < 9\}.$$

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Solution: Let $\tilde{\mathcal{S}}$ be the portion of the sphere parameterized by

$$X(u,v) = (u, v, \sqrt{1/9 - u^2 - v^2})$$

for $1/36 < u^2 + v^2 < 1/9$. Alternatively, this may be expressed as

$$\tilde{\mathcal{S}} = \{(u, v, \sqrt{1/9 - u^2 - v^2}) : 1/36 < u^2 + v^2 < 1/9\}.$$

A diffeomorphism $\phi : \mathcal{S} \to \tilde{\mathcal{S}}$ is given by

$$\phi(x,y,0) = \left(\frac{\sqrt{x^2+y^2+2}}{6} x, \frac{\sqrt{x^2+y^2+2}}{6} y, \ldots \right).$$

6. (25 points) (3-3.20) Find the umbilical points of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Solution: