MATH 6701 Fall 2020
Mathematical Methods in the Physical Sciences I
Introduction

John McCuan

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What I have to say here is fairly simple: This course involves three topics:

1. linear algebra
2. ordinary differential equations (ODE)
3. complex analysis

The first two topics are supposed to be a review. The student is supposed to have previously taken courses devoted to linear algebra and ODEs. The students are probably also expected to have at least a passing knowledge of complex numbers at least as they arise in highschool algebra in solving quadratic equations along with the algebraic manipulations which may be more appropriate for a course in "college algebra" or perhaps some versions of pre-calculus. In particular, one guesses that everyone knows the complex number i is one of two solutions to the quadratic equation

$$x^2 = -1,$$

and with the introduction of this number one can solve all quadratic equations because one can make sense of $\sqrt{d}$ for any real number d. Furthermore, using the property that $i^2 = -1$ and the distributive property (or "F-O-I-L" or whatever you want to call it) from algebra

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i.$$

Generally, one has in mind that a, b, c, and d are real numbers here, but the calculation works in any case. For example,

$$(3+2i)(5-i) = 15+2-3i+10i = 17+7i.$$

You probably know the consequence that a product like

$$(3+2i)(3-2i) = 9+4 = 13$$

is always real and positive. The numbers $a+bi$ and $a-bi$ are called **complex conjugates** when a and b are real. Conjugate multiplication helps one simplify complex fractions like

$$\frac{3+2i}{5-i} = \frac{3+2i}{5-i}\,\frac{5+i}{5+i} = \frac{13(1+i)}{26} = \frac{1}{2}+\frac{1}{2}\,i.$$

It turns out that there is a whole lot more to say about complex numbers, and some good portion of that can be useful in engineering, so it is included in this course. That material will probably be the "most new" to most of you, and often the course is taught in the order listed above. Covering linear algebra first (or at least before ODEs) makes sense because linear algebra is basically quite an easy subject and it is used in some way to solve some ODEs. There is linear structure in the solution sets of linear ODEs. The complex analysis is usually viewed as somewhat independent and is usually tacked on the end. Part of the result of this is that students usually don't learn much complex analysis in 6701. It's harder (because it's new) and it is tacked on at the end, so it seems like it's not important...or everybody is tired or whatever.

I'm going to do things a little differently. Your book covers these topics in, more or less, the following way:

Chapter 2 Complex Numbers

Chapter 3 Linear Algebra

Chapter 8 Ordinary Differential Equations

Chapter 12 Series Solutions of Ordinary Differential Equations

Chapter 14 Functions of a Complex Variable

I'm going to start with complex analysis. I'll try to make it quick and painless. But it's probably going to be painful. The good news is that learning is painful and complex analysis is beautiful. So that's something. Also, we'll get the "hard" part out of the way at the beginning.

I've probably said enough about how the course is going to go. You'll find out more about that soon enough. Two important resources are the course page

`http://www.math.gatech.edu/~mccuan/courses/6701/`

and the assignments page (linked from the course page). But if you're reading this, you've presumably already found those. Maybe the last introductory comment I'll make is that I'll probably follow the textbook in a rather loose manner. Some of the material is in there. I will, no doubt, discuss things that are not in there and discuss some things from a more advanced viewpoint. But it's a good elementary text. I recommend it; you can learn a lot of useful things from it. I like it, especially because it has a chapter on the calculus of variations. Often I've covered some material in this course on the calculus of variations after reviewing ODEs. I probably won't do that this semester. We'll see how it goes.

One more comment: I'm planning to give lectures using video conferencing software. You should be able to access those lectures (MW 12:30-1:45 EST) when they are given and access a recording of them at some later time. You should have the ability to ask questions during the lectures, and that can be helpful to me too. It can also be helpful to me if you answer (or even try to answer) questions that I ask you. Both answering and asking questions help me understand what you know and don't know, so I can adjust my presentation to be more accessible for you (hopefully). So please don't be shy about speaking up. Of course, if everyone starts talking at once, or if one person has a lot of not so helpful stuff to say, things can get confusing and chaotic and a counterproductive situation can be created. Hopefully, each of you can figure out when that is happening and moderate your contribution accordingly, but generally (I think) students err on the side of being too passive and quiet.