Math 6701 Practice Final Exam

Classical Mathematical Methods in the Physical Sciences I

July 24, 2012

Name and section: ____________________

1. (20 points) (Euler-Cauchy-Wilks ODE; 3.6.10) Solve the ODE

$$4x^2y'' + 4xy' - y = 0.$$

2. (20 points) (Second ODE problem; 10.3.4) Use diagonalization to solve the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x}.$$

3. (20 points) (8.6.8) If the matrix

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 11 & 14 \\ -1 & 4 & 7 \end{pmatrix}$$

is invertible, use the formula

$$A^{-1} = \frac{1}{\det A} \left(A^{\text{cof}}\right)^T$$

to find the inverse. If there is no inverse, explain how you know there is no inverse.

4. (20 points) (8.12.4) Determine if the matrix

$$A = \begin{pmatrix} 0 & 5 \\ 1 & 0 \end{pmatrix}$$

is diagonalizable. If A is diagonalizable, find an invertible matrix P such that $P^{-1}AP = D$ is diagonal and compute D.

5. (20 points) (17.3.25) Let $f(z) = z^2 + \bar{z}^2$ Find

$$\{f(z) : \operatorname{Re} z = 2 \text{ and } |z| \leq 4\}.$$

6. (20 points) (17.5.27) Find the harmonic conjugate of $u(x, y) = \ln(x^2 + y^2)$.