1. If $\Omega \subset \text{of } \mathbb{R}^n$ is open and $f \in C^1(\Omega)$, show $f \in C^0(\Omega)$.

2. Finish the in-class proof of equality of mixed partials by showing that

$$
\int_R \frac{\partial^2 f}{\partial x_i \partial x_j} = \int_{\hat{R}_{ij}} [f(x_B) - f(x_A) - f(x_C) + f(x_D)]
$$

where $R = [x_0^1 - \epsilon, x_0^1 + \epsilon] \times \cdots \times [x_0^n - \epsilon, x_0^n + \epsilon]$ and $\hat{R}_{ij}$ is the $n-2$ dimensional rectangle with the $i$ and $j$ factors omitted, and

- $x_B = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_A = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_C = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j - \epsilon)e_j$,
- $x_D = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j - \epsilon)e_j$.

3. Extend the reasoning presented in class to obtain the second order Taylor expansion with remainder for $f \in C^2(\Omega)$:

$$
f(x) = f(x_0) + \langle Df(x_0), v \rangle + \frac{1}{2} \langle D^2f(x_0)v, v \rangle + o(\|v\|^3).
$$

Use your result to give another proof of the second necessary condition for minimizers.

4. Use the reasoning presented in class to prove Taylor’s approximation formula for $g \in C^k[a, b]$. 

---

Calculus of Variations

Homework 2

January 13, 2010

1. If $\Omega \subset \text{of } \mathbb{R}^n$ is open and $f \in C^1(\Omega)$, show $f \in C^0(\Omega)$.

2. Finish the in-class proof of equality of mixed partials by showing that

$$
\int_R \frac{\partial^2 f}{\partial x_i \partial x_j} = \int_{\hat{R}_{ij}} [f(x_B) - f(x_A) - f(x_C) + f(x_D)]
$$

where $R = [x_0^1 - \epsilon, x_0^1 + \epsilon] \times \cdots \times [x_0^n - \epsilon, x_0^n + \epsilon]$ and $\hat{R}_{ij}$ is the $n-2$ dimensional rectangle with the $i$ and $j$ factors omitted, and

- $x_B = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_A = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_C = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j - \epsilon)e_j$,
- $x_D = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j - \epsilon)e_j$.

3. Extend the reasoning presented in class to obtain the second order Taylor expansion with remainder for $f \in C^2(\Omega)$:

$$
f(x) = f(x_0) + \langle Df(x_0), v \rangle + \frac{1}{2} \langle D^2f(x_0)v, v \rangle + o(\|v\|^3).
$$

Use your result to give another proof of the second necessary condition for minimizers.

4. Use the reasoning presented in class to prove Taylor’s approximation formula for $g \in C^k[a, b]$. 

---

Calculus of Variations

Homework 2

January 13, 2010

1. If $\Omega \subset \text{of } \mathbb{R}^n$ is open and $f \in C^1(\Omega)$, show $f \in C^0(\Omega)$.

2. Finish the in-class proof of equality of mixed partials by showing that

$$
\int_R \frac{\partial^2 f}{\partial x_i \partial x_j} = \int_{\hat{R}_{ij}} [f(x_B) - f(x_A) - f(x_C) + f(x_D)]
$$

where $R = [x_0^1 - \epsilon, x_0^1 + \epsilon] \times \cdots \times [x_0^n - \epsilon, x_0^n + \epsilon]$ and $\hat{R}_{ij}$ is the $n-2$ dimensional rectangle with the $i$ and $j$ factors omitted, and

- $x_B = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_A = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j + \epsilon)e_j$,
- $x_C = x - (x_i e_i + x_j e_j) + (x_0^i - \epsilon)e_i + (x_0^j - \epsilon)e_j$,
- $x_D = x - (x_i e_i + x_j e_j) + (x_0^i + \epsilon)e_i + (x_0^j - \epsilon)e_j$.

3. Extend the reasoning presented in class to obtain the second order Taylor expansion with remainder for $f \in C^2(\Omega)$:

$$
f(x) = f(x_0) + \langle Df(x_0), v \rangle + \frac{1}{2} \langle D^2f(x_0)v, v \rangle + o(\|v\|^3).
$$

Use your result to give another proof of the second necessary condition for minimizers.

4. Use the reasoning presented in class to prove Taylor’s approximation formula for $g \in C^k[a, b]$.