Calculus of Variations
Homework 2

January 30, 2012

1. Find the potential function for an inverse square gravitational field.
2. Formulate the variational problem associated with Newton's second law for a path of motion $\mathbf{x} : [0, T] \to \mathbb{R}^3$. (And derive Newton's law.)
3. Find the Euler-Lagrange ODE for the following functionals:
   (a) $\int_0^1 y' \, dx$.
   (b) $\int_0^1 yy' \, dx$.
   (c) $\int_0^1 xyy' \, dx$.
   (d) $\int_0^1 y'^2/x^3 \, dx$.
   (e) $\int_0^1 (y^2 + y'^2 + 2ye^x) \, dx$.
   (f) $\int_0^1 \sin y' \, dx$.
   (g) $\int_0^1 \sqrt{1 + y'^2}/x \, dx$.
   (h) There are more in problem 4 of section 2.3 of Sagan. There is also a discussion of the Euler-Lagrange equation in section 2.5.
4. How would your answer in the previous problems change if the integral were taken over a different interval?
5. See if you can solve the Euler-Lagrange equations you found above in $\mathcal{A} = \{y \in C^2[0, 1] : y(0) = 0, y(1) = 1\}$.