# Calculus of Variations
# Homework 5

March 6, 2012

1. Consider the area functional on the admissible set

$$\{u \in C^1[a,b] : u(a) = r_1, u(b) = r_2\}$$

obtained by rotating the graph of $u$ about the axis containing the domain $[a,b]$.

   (a) Solve the Euler-Lagrange equation. Fix a particular extremal and determine the conditions under which you can embed it in a field of extremals.

   (b) Find a particular extremal which can be embedded in a field of extremals, determine the associated tangent field, and write down the Eikonal/Carathéodory equations for the field.

   (c) Determine which extremals are local minima.

2. Given a smooth Lagrangian $F = F(x,z,p)$ show that the mapping $p \mapsto F(x,z,p)$ is strictly convex on $\mathbb{R}^n$ (for fixed $(x,z)$) if $F$ satisfies the *ellipticity condition*: There is some $\epsilon > 0$ such that

$$\sum_{i,j=1}^{n} F_{p_i p_j}(x,z,p)\xi_i \xi_j \geq \epsilon |\xi|^2$$

for all $\xi \in \mathbb{R}^n$.