

Calculus of Variations

Homework 6

March 7, 2012

1. Give a derivation of Newton's expression for the drag on a rotationally symmetric profile given by $u = u(r)$ moving through a fluid:

$$\mathcal{D}[u] = \int_0^R \frac{r}{1 + u'(r)^2} dr.$$

2. Numerically render some rotationally symmetric profiles determined by

$$u(r) = M \sin^2(hr)$$

and show that for these profiles

$$\lim_{h \rightarrow +\infty} \mathcal{D}[u] = 0.$$

3. Investigate numerically the family of catenoids determined as candidates for minimal area surfaces of rotation.
4. Verify the sufficient conditions for at least some catenoid to be a local minimum including the condition that there are no Jacobi fields.