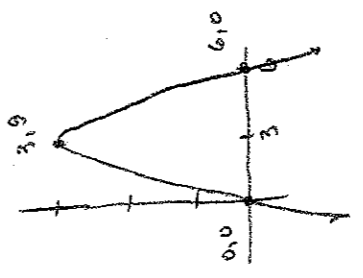
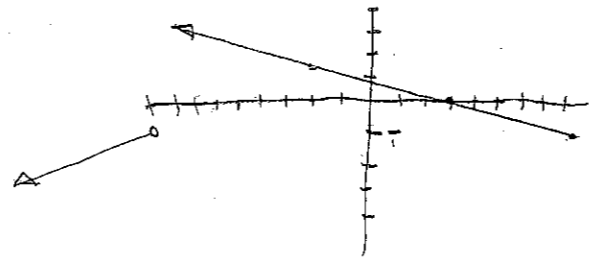
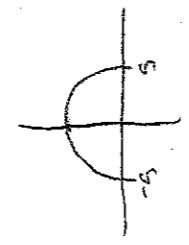
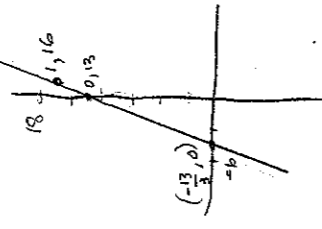
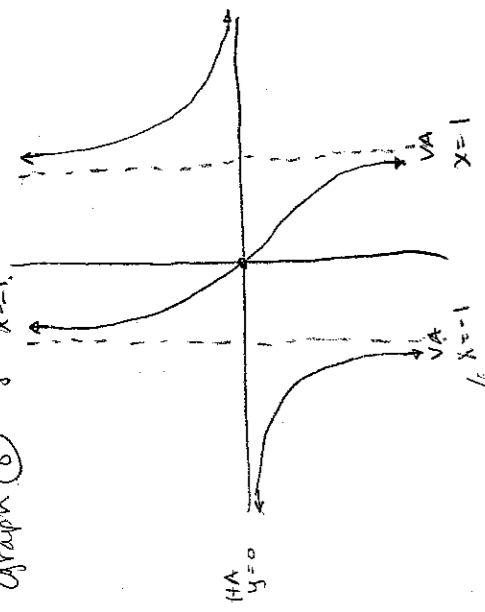


Graph 8  $y = \frac{x}{x^2-1}$



factor picture:

$x$	-	+	+
$x-1$	-	+	+
$x+1$	-	+	+

N<sup>-</sup> P<sup>0</sup> N<sup>+</sup> P

FOR GRAPH, SEE PAGE TOP

factor picture

$2x^3$	-	+	+
$x^2+1$	-	+	+
$x^2+1$	-	+	+

N<sup>+</sup> O P

always positive

Graph 9  $y = \frac{2x^3}{x^2+1}$

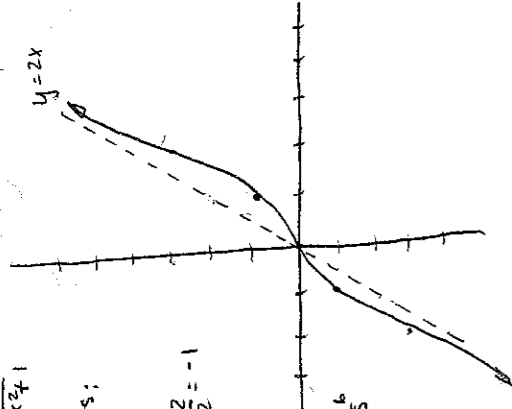
plot some points:

$x=1$   $y = \frac{2(1)^3}{1^2+1} = 1$

$x=-1$   $y = \frac{2(-1)^3}{(-1)^2+1} = \frac{-2}{2} = -1$

$x=2$   $y = \frac{2(2)^3}{2^2+1} = \frac{16}{5}$

$x=-2$   $y = \frac{2(-2)^3}{(-2)^2+1} = \frac{-16}{5}$



29 April

PreCalculus - Final Review

1. Find the slope and y-intercept (if possible) of the equation of the line. Sketch the line:

$y = 3x + 13$   $y = mx + b$   $m = 3$  slope  $b = 13$   $y$  int.

$m = 3 =$  slope  $b = 13$   $y$  int =  $(0, 13)$

$y = 0 = 3x + 13$   $3x = -13$   $x = -\frac{13}{3}$

2. Determine the domain of the function. Verify your result with a graph.

$f(x) = \sqrt{25-x^2}$  no negative even root

$25-x^2 \geq 0$   $(5-x)(5+x) \geq 0$

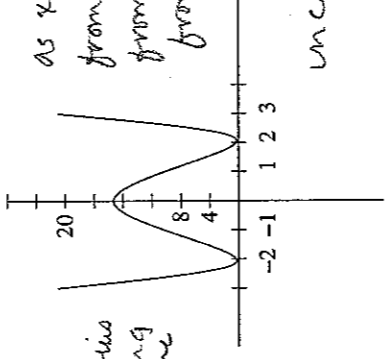
$5-x$	pos	pos	neg
$5+x$	neg	pos	pos
	N	-	P
	S	S	N

so  $-5 \leq x \leq 5$  N [-5, 5]

$y = \sqrt{25-x^2}$   
 $y^2 = 25-x^2$   
 $x^2+y^2 = 25$   
 upper half circle

3. Determine the intervals over which the function is increasing, decreasing or constant.

as  $x$  moves to  $-2$ ,  $y$ -values falling  
 from  $-2$  to  $0$ ,  $y$  values rising  
 from  $0$  to  $2$ ,  $y$  values falling  
 from  $2$  on,  $y$ -values rising



increasing:  $-2 < x < 0$  and  $x > 2$

decreasing:  $x < -2$  and  $0 < x < 2$

4. Graph the function:  $f(x) = \begin{cases} 5x-3, & x \geq -1 \\ -4x+5, & x < -1 \end{cases}$

$y = 4 - \frac{1}{3}x$   $x = 4 - \frac{1}{3}y$   $(x-4) = -\frac{1}{3}y$   $y = -3(x-4) = -3x+12$

5. Find the inverse. Verify:  $f(x) = 4 - \frac{1}{3}x$

$f(f^{-1}(x)) = f(-3x+12) = 4 - \frac{1}{3}(-3x+12) = 4 + \frac{2}{3}x - \frac{12}{3} = 4 + \frac{2}{3}x - 4 = \frac{2}{3}x$

$f^{-1}(f(x)) = f^{-1}(4 - \frac{1}{3}x) = -3(4 - \frac{1}{3}x) + 12 = -12 + x + 12 = x$

6. Write the quadratic function in standard form and sketch its graph. Identify the vertex and x-intercepts.

$f(x) = 6x - x^2$  complete square

$f(x) = -x^2 + 6x = -(x^2 - 6x + 9) + 9 = -(x-3)^2 + 9$

parabola opens DOWN because  $a = -1$  vertex at (3, 9)

$x$  intercepts:  $0 = 6x - x^2 = x(6-x)$   $x = 0$   $x = 6$

7. Find the domain of the rational function  $f(x) = \frac{8}{x^2 - 10x + 24}$  no division by zero.

check when denominator = 0  $(x-6)(x-4) = 0$

$x^2 - 10x + 24 = 0$   $x = 6$   $x = 4$

8. Identify intercepts, check for symmetry, identify any vertical or horizontal asymptotes, and sketch the graph of the rational function.

$y = \frac{x}{x^2-1}$   $x$ -int means  $y = 0$   $0 = \frac{x}{x^2-1}$  fraction zero only when  $y$ -int (0,0)

vertical asymptotes where denominator = 0  $(x^2-1) = 0$   $(x-1)(x+1) = 0$   $x = -1$   $x = 1$

deg num < deg den so  $y = 0$  horizontal asymptote

9. State the domain of the function and identify any vertical and slant asymptotes. Then sketch the graph of the rational function.

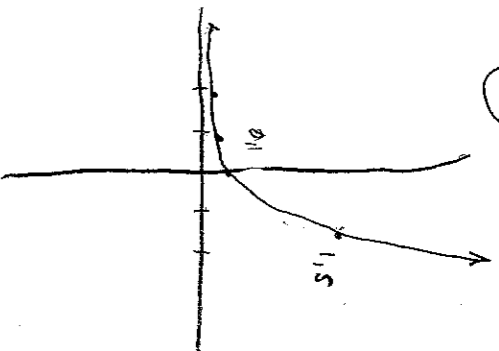
$f(x) = \frac{2x^3}{x^2+1}$   $x = 0$   $y = 0$   $x^2+1 \neq 0$  deg num > deg den so slant asymptote. Find by long division

$2x^3 \div (x^2+1) = 2x - \frac{2x}{x^2+1}$

$f(x) = 2x - \frac{2x}{x^2+1}$  so slant asymptote  $y = 2x$

10

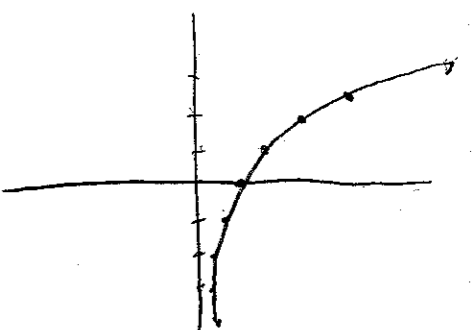
x	5 <sup>x</sup>
-2	5 <sup>-2</sup> = 1/25
-1	5 <sup>-1</sup> = 1/5
0	5 <sup>0</sup> = 1
1	5 <sup>1</sup> = 5
2	5 <sup>2</sup> = 25
3	5 <sup>3</sup> = 125



10. Graph:  $f(x) = 5^x$

11

x	(2/3) <sup>x</sup>
-3	(2/3) <sup>-3</sup> = 27/8
-2	(2/3) <sup>-2</sup> = 9/4
-1	(2/3) <sup>-1</sup> = 3/2
0	(2/3) <sup>0</sup> = 1
1	(2/3) <sup>1</sup> = 2/3
2	(2/3) <sup>2</sup> = 4/9
3	(2/3) <sup>3</sup> = 8/27



11. Graph:  $f(x) = \left(\frac{2}{3}\right)^x$

Compound Interest: Complete the table to determine the balance  $A$  for  $P$  dollars invested at rate  $r$  for  $t$  years and compounded  $n$  times per year.

n	1	2	3	4	365	Continuous
A						

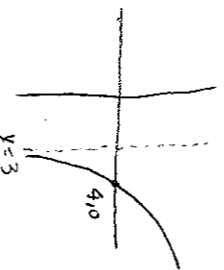
$n = 1$   
 $A = 3500(1 + 0.065)^{10}$   
 $= 3500(1.065)^{10}$   
 $\approx 3500(1.8171)$   
 $\approx \$6,569.98$

$n = 2$   $A = 3500(1 + \frac{0.065}{2})^{20} = 3500(1.0325)^{20} \approx 3500(1.896) = \$6,635.42$   
 $n = 3$   $A = 3500(1 + \frac{0.065}{3})^{30} = 3500(1.0216)^{30} \approx 3500(1.899) = \$6,644.99$

12.  $P = \$3500, r = 6.5\%, t = \frac{5}{3} = 10$  years  
 $n = 4$   $A = 3500(1 + \frac{0.065}{4})^{40} = 3500(1.01625)^{40} \approx 3500(1.91) = \$6,669.46$   
 $n = 365$   $A = 3500(1 + \frac{0.065}{365})^{3650} \approx 3500(1.000178)^{3650} \approx 3500(1.91543) = \$6,704.00$

$A = 3500 e^{0.065(10)} = 3500 e^{0.65} \approx 3500(1.91554) = \$6,704.39$

13. Find the domain,  $x$ -intercept, and vertical asymptote of the logarithmic function and sketch its graph.  
 $f(x) = \ln(x-3)$        $y < \ln 4$       Domain  $\ln x$  is  $x > 0$

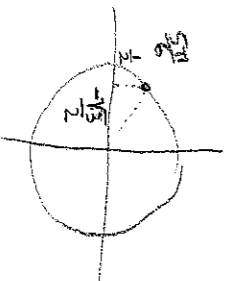


14. Condense the expression to the logarithm of a single quantity.  
 $5 \ln(x-2) - \ln(x+2) - 3 \ln x = \ln(x-2)^5 - \ln(x+2) - \ln x^3$   
 $= \ln(x-2)^5 - [\ln(x+2) + \ln x^3] = \ln(x-2)^5 - \ln(x+2)x^3$   
 $= \ln\left(\frac{(x-2)^5}{(x+2)x^3}\right)$

15. Solve for  $x$ :       $3^x = 729$        $729 = 9 \cdot 81 = 9 \cdot 9 \cdot 9 = 3^2 \cdot 3^2 \cdot 3^2 = 3^6$   
 Get same base, then exponents equal  
 $3^x = 3^6$        $x = 6$

16. Solve the logarithmic equation. Approximate the result to three decimal places.  
 $\ln x - \ln 3 = 2$        $\ln\left(\frac{x}{3}\right) = 2$        $e^{\ln(\frac{x}{3})} = e^2$        $\frac{x}{3} = e^2$   
 $x = 3e^2 \approx 22.167$

17. Find the point  $(x, y)$  on the unit circle that corresponds to the real number  $t$ .  
 $t = \frac{5\pi}{6}$        $(x, y) = \left(\cos \frac{5\pi}{6}, \sin \frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

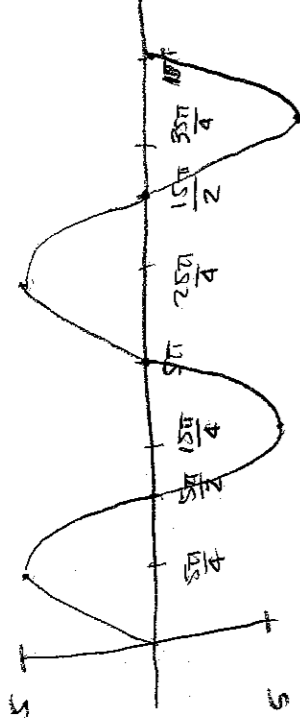


18. Evaluate (if possible) the six trigonometric functions of the real number.

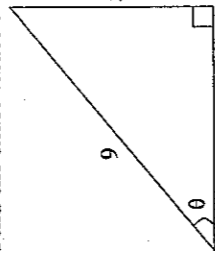
$t = \frac{\pi}{4}$   
 $\sin t = \frac{1}{2}$        $\tan t = 1$        $\sec t = \sqrt{2}$   
 $\cos t = \frac{1}{\sqrt{2}}$        $\cot t = 1$        $\csc t = \sqrt{2}$



Graph (22)



19. Find the exact values of the six trigonometric functions of the angle  $\theta$  shown in the figure.

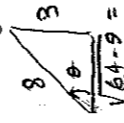


$$\begin{aligned} \sin \theta &= \frac{3}{5} & \cot \theta &= \frac{4}{3} \\ \cos \theta &= \frac{4}{5} & \sec \theta &= \frac{5}{4} \\ \tan \theta &= \frac{3}{4} & \csc \theta &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \sqrt{9^2 - 8^2} &= \sqrt{81 - 64} \\ &= \sqrt{17} \end{aligned}$$

20. Find the remaining five trigonometric functions of  $\theta$  satisfying the condition.

$$\sin \theta = \frac{3}{8}$$



$$\begin{aligned} \cos \theta &= \frac{\sqrt{55}}{8} & \cot \theta &= \frac{\sqrt{55}}{3} & \csc \theta &= \frac{8}{3} \\ \tan \theta &= \frac{3}{\sqrt{55}} & \sec \theta &= \frac{8}{\sqrt{55}} \end{aligned}$$

21. Evaluate the sine, cosine, and tangent of the angle without using a calculator:  $\frac{\pi}{3}$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \cos \frac{\pi}{3} = \frac{1}{2} \quad \tan \frac{\pi}{3} = \sqrt{3}$$

22. Sketch a graph of the function. Include two full periods.

$$f(x) = 5 \sin \frac{2x}{5} \quad \text{amplitude } 5 \quad \text{"starting point" } 0 \quad \text{new } \text{pd} = \frac{2\pi}{\frac{2}{5}} = 5\pi$$

23. Name the trigonometric function that is equivalent to the expression:  $\sqrt{1 + \tan^2 x}$

$$\frac{\sec^2 x + \cos^2 x}{\cos^2 x} \Rightarrow \tan^2 x + 1 = \sec^2 x \quad \sqrt{1 + \tan^2 x} = \sec x$$

24. Use the fundamental trigonometric identities to simplify the trigonometric expression:

$$\begin{aligned} \frac{\tan^2 \theta \csc^2 \theta - \tan^2 \theta}{\tan^2 \theta \csc^2 \theta - \tan^2 \theta} &= \frac{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} - 1}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} - 1} \\ \tan^2 \theta (\csc^2 \theta - 1) &= 1 \\ \tan^2 \theta (\cot^2 \theta) &= 1 \end{aligned}$$

25. Verify the identity:  $\sec^2 x \cot x - \cot x = \tan x$

$$\begin{aligned} (\sec^2 x - 1) \cot x &\stackrel{?}{=} \tan x \\ \tan^2 x \cot x &\stackrel{?}{=} \tan x \\ \tan x \tan x \cot x &\stackrel{?}{=} \tan x \end{aligned}$$

26. Solve the equation:  $4 \cos \theta = 1 + 2 \cos \theta$

$$\begin{aligned} 4 \cos \theta &= 1 + 2 \cos \theta \\ -2 \cos \theta &= 1 \\ 2 \cos \theta &= -1 \\ \cos \theta &= -\frac{1}{2} \\ \theta &= \frac{2\pi}{3}, \frac{4\pi}{3} \end{aligned}$$

27. Solve the system by the method of substitution:

$$\begin{cases} x = y + 3 \\ x = y^2 + 1 \end{cases} \quad \begin{aligned} y + 3 &= x = y^2 + 1 \\ y + 3 &= y^2 + 1 \\ 0 &= y^2 - y - 2 \end{aligned} \quad \begin{aligned} 0 &= (y-2)(y+1) & x &= 2 + 3 = 5 \\ y &= 2 \quad y = -1 & x &= -1 + 3 = 2 \end{aligned} \quad \begin{aligned} & & & (5, 2) \\ & & & (2, -1) \end{aligned}$$

28. Solve the system by elimination:

$$\begin{cases} 2x - y = 2 \\ 6x + 8y = 39 \end{cases} \quad \begin{aligned} -6x + 3y &= -6 \\ 6x + 8y &= 39 \\ \hline 11y &= 33 \\ y &= 3 \end{aligned} \quad \begin{aligned} 2x - 3 &= 2 \\ 2x &= 5 \\ x &= \frac{5}{2} \end{aligned}$$

$$\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & -6 & \\ 2 & -3 & 0 & 0 & -7 & \\ -1 & 3 & -3 & 1 & 11 & \end{array}$$

$$\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & -6 & \\ 0 & -1 & 2 & 1 & -5 & \\ 0 & 1 & -2 & 1 & 5 & \end{array}$$

$$\begin{aligned} -y + 2z &= -5 \\ -y + 2z &= -5 \\ y - 2z &= 5 \\ y &= 5 + 2z \end{aligned}$$

Problem 30

$$\begin{array}{ccc|ccc} 2r_1 - r_2 - 0r_3 & & & & & \\ 1 & 2 & -4 & 2 & -12 & \\ -2 & 3 & 0 & 0 & 7 & \\ 0 & -1 & 2 & -2 & -5 & \end{array}$$

$$\begin{array}{ccc|ccc} r_2 + r_3 - 0r_3 & & & & & \\ 0 & -1 & 2 & 0 & -5 & \\ 0 & 1 & -2 & 2 & 5 & \\ 0 & 0 & 0 & 0 & 0 & \end{array}$$

$$\begin{aligned} x - 2(5 + 2z) + t &= -6 \\ x - 10 - 4t + t &= -6 \\ x - 10 - 3t &= -6 \\ x &= 4 + 3t \end{aligned}$$

infinite solutions

$$\begin{aligned} x - 10 - 3t &= -6 \\ x &= 4 + 3t \end{aligned}$$

29. Use back-substitution to solve the system:

$$\begin{aligned} z &= -5 \\ -y + (-5) &= -1 \\ -y &= 4 \\ y &= -4 \end{aligned}$$

$$\begin{aligned} x - 4(-4) + 3(-5) &= 3 \\ x + 16 - 15 &= 3 \\ x + 1 &= 3 \\ x &= 2 \end{aligned}$$

(2, -4, -5)

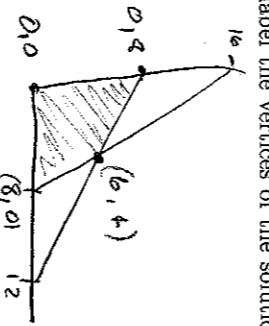
30. Use Gaussian elimination to solve the system of equations:

$$\begin{cases} x - 2y + z = -6 \\ 2x - 3y = -7 \\ -x + 3y - 3z = 11 \end{cases}$$

see top

$$\begin{cases} x - 4y + 3z = 3 \\ -y + z = -1 \\ z = -5 \end{cases}$$

$$\begin{cases} 2x + 3y \leq 24 \\ 2x + y \leq 16 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



$$\begin{aligned} 2x + 3y &= 24 \\ -2x - y &= -16 \\ \hline 2y &= 8 \\ y &= 4 \\ x &= 6 \end{aligned}$$

31. Sketch a graph and label the vertices of the solution set of the system of inequalities:

$$\begin{cases} 2x + y + 2z = 1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases}$$

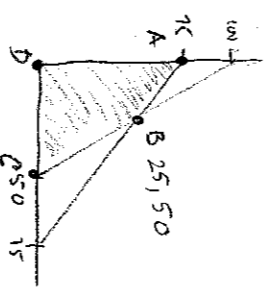
$$\begin{aligned} M &= 0 \text{ at } (0,0) \\ M &= 600 \text{ at } (25,50) \end{aligned}$$

$$\begin{aligned} A &= (0, 75) & z &= 10(0) + 7(75) = 525 \\ B &= (25, 50) & z &= 10(25) + 7(50) = 600 \\ C &= (50, 0) & z &= 10(50) + 7(0) = 500 \\ D &= (0, 0) & z &= 10(0) + 7(0) = 0 \end{aligned}$$

32. Sketch the region determined by the constraints. Then find the minimum and maximum values of the objective function and where they occur, subject to the indicated restraints.

$$\begin{aligned} \text{Objective function: } z &= 10x + 7y \\ \text{Constraints: } x &\geq 0 \\ &y \geq 0 \end{aligned}$$

check vertices:



$$\begin{aligned} 2x + y &= 100 \\ -x - y &= -75 \\ \hline x &= 25 \end{aligned}$$

33. Use matrices to solve the system of equations. Use Gauss-Jordan elimination:

$$\begin{cases} -x + y + 2z = 1 \\ 2x + 3y + z = -2 \\ 5x + 4y + 2z = 4 \end{cases}$$

$$\begin{bmatrix} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{bmatrix}$$

(see top of page 5)

34. If possible, find (a)  $A+B$ ; (b)  $A-B$ ; (c)  $4A$ ; and (d)  $A+3B$ .

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 2-3 & -2+10 \\ 3+12 & 5+8 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 15 & 13 \end{bmatrix}$$

$$3B = \begin{bmatrix} 3(-3) & 3(10) \\ 3(12) & 3(8) \end{bmatrix} = \begin{bmatrix} -9 & 30 \\ 36 & 24 \end{bmatrix}$$

35. Find  $AB$  if possible

$$A = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 12 \\ 20 & 40 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5(4) + 4(20) & 5(12) + 4(40) \\ -7(4) + 2(20) & -7(12) + 2(40) \\ 11(4) + 2(20) & 11(12) + 2(40) \end{bmatrix} = \begin{bmatrix} 100 & 220 \\ 12 & -2 \\ 84 & 212 \end{bmatrix}$$

Perform the matrix operations. If it is not possible, explain why.

36.  $\begin{bmatrix} 1 & 5 & 6 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix}$   $\begin{matrix} 2 \times 3 \\ 2 \times 3 \end{matrix}$   $\begin{matrix} 4 \\ 4 \end{matrix}$   $\begin{matrix} \text{not possible, matrices are not} \\ \text{conformable} \end{matrix}$

Doesn't match

$$\begin{array}{r} 33 \\ \hline -1 \ 1 \ 2 \ 1 \ 1 \\ 2 \ 3 \ 1 \ -2 \\ 5 \ 4 \ 2 \ 4 \ 3 \end{array} \quad \begin{array}{r} -1 \ 1 \ 2 \ 1 \ 1 \\ 0 \ 1 \ 1 \ 0 \\ 0 \ 0 \ 1 \ 3 \end{array}$$

$$\begin{array}{r} 2r_1 + r_2 \rightarrow r_3 \\ \hline -2 \ 2 \ 4 \ 2 \\ 2 \ 3 \ 1 \ -2 \\ \hline 0 \ 5 \ 5 \ 0 \Rightarrow 0 \ 1 \ 1 \ 0 \\ \hline 0 \ 0 \ -1 \ -3 \end{array}$$

$$\begin{array}{r} 5r_1 + r_3 \rightarrow r_3 \\ \hline -5 \ 5 \ 10 \ 5 \\ 5 \ 4 \ 2 \ 4 \\ \hline 0 \ 9 \ 12 \ 9 \Rightarrow 0 \ 3 \ 4 \ 3 \end{array}$$

$$\begin{aligned} z &= 3 \\ y + 3 &= 0 \\ y &= -3 \\ -x + (-3) + 2(3) &= 1 \\ -x + 3 &= 1 \\ -x &= -2 \\ x &= 2 \end{aligned} \quad (2, -3, 3)$$

$$37. \begin{bmatrix} 4 & -2 & 6 \\ 2 & 0 & -3 \\ 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 0 \end{bmatrix} = [4(-2) + 2(6) + 6(0)] = [4(-2) + 2(-3) + 6(0)] = [-8 + 0 + 12 \quad 4 + 6 + 0] = [4 \ 10]$$

$$38. \text{ Show that } B \text{ is the inverse of } A. \quad A = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} \quad AB = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} -10 + 11 & 5 - 5 \\ -22 + 22 & 11 - 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

39. Find the inverse of the matrix (if it exists).

$$\begin{bmatrix} -3 & -5 \\ 2 & 3 \end{bmatrix} \begin{array}{l} -3 \ -5 \ 1 \ 0 \\ 2 \ 3 \ 0 \ 1 \end{array} \quad \begin{array}{l} 2r_1 + 3r_2 \rightarrow r_2 \\ -6 \ -10 \ 2 \ 0 \\ \hline 6 \ 9 \ 0 \ 3 \\ \hline 0 \ -1 \ 2 \ 3 \end{array} \quad \begin{array}{l} -3 \ -5 \ 1 \ 0 \\ 0 \ -1 \ 2 \ 3 \\ \hline 0 \ 5 \ -10 \ -15 \\ \hline 0 \ 5 \ -10 \ -15 \end{array} \quad A^{-1} = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$$

40. Use an inverse matrix to solve (if possible) the system of linear equations.

$$\begin{cases} -x + 4y = 8 \\ 2x - 7y = -5 \end{cases} \quad A^{-1} = \frac{1}{(-1)(-7) - 2(4)} \begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -7 & -4 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} \quad A^{-1}b = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \end{bmatrix} \quad x = 36 \quad y = 11$$

41. Use the Binomial Theorem to expand the binomial. Simplify your answer.

$$(3x + y^2)^7 = 1(3x)^7 + 7(3x)^6(y^2) + 21(3x)^5(y^2)^2 + 35(3x)^4(y^2)^3 + 35(3x)^3(y^2)^4 + 21(3x)^2(y^2)^5 + 7(3x)(y^2)^6 + (y^2)^7$$

42. Find the center, vertices, foci, and eccentricity of the ellipse.

$$\frac{(x+2)^2}{81} + \frac{(y-1)^2}{100} = 1 \quad \begin{array}{l} \frac{9^2}{a^2} \\ \frac{10^2}{b^2} \end{array} \quad \text{Center } (-2, 1) \\ \text{Major } (y) \quad (-2, 11) \text{ \& } (-2, -9) \\ \text{Minor } (x) \quad (-11, 1) \text{ \& } (7, 1) \end{array}$$

$$\text{foci: } \begin{array}{l} a^2 = b^2 + c^2 \\ 100 = 81 + c^2 \\ c^2 = 9 \\ c = \pm 3 \\ (-2, 4) \text{ \& } (-2, -2) \end{array}$$

43. Find the center, vertices, foci, and the equations of the asymptotes of the hyperbola, and sketch its graph.

$$\frac{(x-3)^2}{16} - \frac{(y+5)^2}{4} = 1 \quad \begin{array}{l} \frac{4^2}{a^2} \\ \frac{2^2}{b^2} \end{array}$$

Slope asymptotes:

$$y = k \pm \frac{b}{a}(x-h)$$

$$y = -5 + \frac{1}{2}(x-3)$$

$$y = -5 - \frac{1}{2}(x-3)$$

Center (3, -5) opens left/right

Vertices: (-1, -5) \& (7, -5)

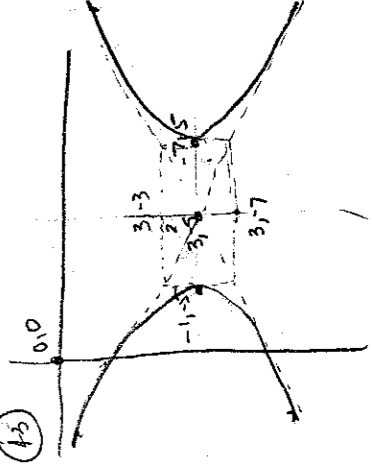
Focus:  $c^2 = a^2 + b^2 \quad c^2 = 16 + 4 = 20 \quad c = \pm 2\sqrt{5} \approx \pm 4.5$

(3 - 2\sqrt{5}, -5) \& (3 + 2\sqrt{5}, -5) \approx (-1.5, -5) \& (7.5, -5)

44. Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola.

$$-4y^2 + 5x + 3y + 7 = 0$$

43



$A = 0 \quad C = -4$   
∴ parabola

