

**MATH 4305, Fall 2015,  
Midterm 2, Practice**

Show all your work. You may use one side of a letter-size sheet of paper for formulae in this exam. Calculator is not allowed. Please give yourself 50 minutes.

**Problem 1** Let  $\mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ , and  $W = \text{span}\{\mathbf{u}, \mathbf{v}\}$ .

a) Find the orthogonal projection of  $\mathbf{y}$  onto  $W$ .

b) Find the distance between  $\mathbf{y}$  and  $W$ .

**Problem 2** Find the trigonometric function of the form  $f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$  that best fits the data points  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 2)$ ,  $(3, 3)$ , using least squares. Compute the least square error. (Remark: This is a problem for concept, find the formula, don't have to solve for exact solution. The test problem will be easier to solve.)

**Problem 3** Find all possible values of  $a$  so that the columns of  $A$  given below are linearly dependent?

$$\begin{pmatrix} a & 2a & 0 & 0 \\ 0 & 0 & a-3 & 3(a-3) \\ 0 & -2a & 0 & 1 \\ 0 & 0 & a-2 & 2(a-2) \end{pmatrix}$$

**Problem 4** (a) Prove that the set  $\mathbf{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$  is a basis for  $\mathbf{P}_2$ .

b) Find the matrix of the linear transformation  $T(f(t)) = f' - 3f$  from  $\mathbf{P}_2$  to  $\mathbf{P}_2$  with respect to the basis  $\mathbf{B}$  found in part (a).

**Problem 5.** Let  $A$  be the following matrix

$$\begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}$$

a) Find the  $QR$  factorization of  $A$ .

b) Find the orthogonal projection of  $\mathbf{b} = (1, 2, 3, 4)^T$  onto  $\text{Col}(A)$ .

**Problem 6:** If  $A$  is an  $n \times n$  matrix, is it true that  $\det(AA^T) = \det(A^T A)$ ?  
Why?