

**MATH 2551, Fall 2017**  
**Practice Exam 1, Chapter 12 and 13**

**Guideline:** Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a sheet of paper (letter size) for formulae, calculator is NOT allowed in this exam. You have 50 minutes.

**Problem 1.** Calculations.

(a)  $\frac{d}{dt}[(2t\mathbf{i} + \sqrt{t}\mathbf{j}) \bullet (t\mathbf{i} - 3\mathbf{j})]$

(b)  $\frac{d}{dt}[(\cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})]$

(c)  $\frac{d}{dt}[e^{\cos 2t}\mathbf{i} + \ln(1 + t^2)\mathbf{j} + (1 - \cos t)\mathbf{k}]$

**Problem 2** A golf ball is hit at time  $t = 0$ . Its position vector as a function of time is given by

$$\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} + (-t^2 + 4t)\mathbf{k}.$$

Notice that at  $t = 0$  the ball is at the origin of the coordinate system. The  $xy$  plane represents the ground. At some time  $t_1 > 0$  the ball will return to the  $xy$  plane at some point  $P(a, b, 0)$ .

(a) Compute the velocity, the acceleration and the speed of the ball at an arbitrary time  $t$ .

(b) Find the time  $t_1 > 0$  and the coordinates of the point  $P$  where the ball hits the  $xy$  plane again.

(c) Set up a definite integral equal to the length of the arc of the trajectory from the origin to the point  $P$ . You do not have to evaluate the integral.

(d) Find the equation of the line tangent to the trajectory at  $P$ .

(e) Find the equation of the vertical plane containing the trajectory.

(f) Find the curvature of the trajectory at  $P$ .

**Problem 3** At each point  $P(x(t), y(t), z(t))$  of its motion, an object of mass  $m$  is subject to a force:

$\mathbf{F}(t) = m\pi^2[4\cos(\pi t)\mathbf{i} + 3\sin(\pi t)\mathbf{j}]$ . Given that  $\mathbf{v}(0) = -3\pi\mathbf{j} + \mathbf{k}$ , and  $\mathbf{r}(0) = 3\mathbf{j}$ . find the following:

(a) The velocity  $\mathbf{v}(1)$ .

(b) The speed  $v(1)$ .

(c) The momentum  $\mathbf{p}(1)$ .

(d) The angular momentum  $\mathbf{L}(1)$ .

(e) The torque  $\tau(1)$ .

(f) The position  $\mathbf{r}(1)$ .

(g) The osculating plane equation at  $\mathbf{r}(1)$ .

(h) The tangential and normal components of acceleration  $\mathbf{a}(1)$ .