MATH 2551, Fall 2018
Practice Exam 1, Chapter 12 and 13

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a sheet of paper (letter size) for formulae, calculator is NOT allowed in this exam. You have 50 minutes.

Problem 1. Calculations.

(a) $\frac{d}{dt}[(2t\mathbf{i} + \sqrt{t}\mathbf{j}) \bullet (t\mathbf{i} - 3\mathbf{j})]$

(b) $\frac{d}{dt}[(cost\mathbf{i} + sint\mathbf{j} + t\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})]$

(c) $\frac{d}{dt}[e^{cos2t}\mathbf{i} + ln(1 + t^2)\mathbf{j} + (1 - cost)\mathbf{k}]$

Problem 2 A golf ball is hit at time $t = 0$. Its position vector as a function of time is given by

$$\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} + (-t^2 + 4t)\mathbf{k}.$$

Notice that at $t = 0$ the ball is at the origin of the coordinate system. The xy plane represents the ground. At some time $t_1 > 0$ the ball will return to the xy plane at some point $P(a, b, 0)$.

(a) Compute the velocity, the accelaration and the speed of the ball at an arbitrary time t.

(b) Find the time $t_1 > 0$ and the coordinates of the point P where the ball hits the xy plane again.

(c) Set up a definite integral equal to the length of the arc of the trajectory from the origin to the point P. You do not have to evaluate the integral.

(d) Find the equation of the line tangent to the trajectory at P.

(e) Find the equation of the vertical plane containing the trajectory.

(f) Find the curvature of the trajectory at P.

Problem 3 At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force:

$\mathbf{F}(t) = m\pi^2[4cos(\pi t)\mathbf{i} + 3sin(\pi t)\mathbf{j}]$. Given that $\mathbf{v}(0) = -3\pi\mathbf{j} + \mathbf{k}$, and $\mathbf{r}(0) = 3\mathbf{j}$. find the following:

(a) The velocity $\mathbf{v}(1)$.

(b) The speed $v(1)$.

(c) The momentum $\mathbf{p}(1)$.

(d) The angular momentum $\mathbf{L}(1)$.

(e) The torque $\tau(1)$.

(f) The position $\mathbf{r}(1)$.

(g) The osculating plane equation at $\mathbf{r}(1)$.

(h) The tengential and normal components of accelaration $\mathbf{a}(1)$.