MATH 2403, Spring 2013
Practice Exam 1, 1.1–2.7

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. You have 50 minutes.

Problem 1. For following equations, state the type of equation and then solve it, explicitly if possible and implicitly otherwise.

(a) $2xy' + y = 10\sqrt{x}$.

(b) $\frac{dy}{dx} = 2xy^2 + 3x^2y^2$

(c) $6xy^3 + 2y^4 + (9x^2y^2 + 8xy^3)y' = 0$

Problem 2. Consider the initial value problem

$$dy/dx = x^2 - y^2, \ y(0) = 1.$$

(a) How many solutions are there solving this initial value problem? How do you know? Verify your answer using the proper theorem.

(b) Approximate $y(0.2)$ using Euler's method with step size 0.1. Show your steps.

Problem 3 A tank with 200 liters volume is full of pure water initially. A mixture containing a concentration of 6 gram/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of salt in the tank at time t. How much salt is in the tank after 120 minute? What is the limiting amount of salt in the tank as $t \to \infty$?

Problem 4 A chemical reaction can be modeled by the following equation

$$dx/dt = \alpha(p - x)(q - x).$$

Where α, and p and q are positive constants.

(a) If $p < q$, find the equilibrium solutions for this equation. Indicate the stable and unstable equilibrium on the phase diagram.

(b) If $x(0) = \frac{p+q}{2}$, $p < q$, determine the limiting value of $x(t)$ as $t \to +\infty$ without solving the differential equation.

(c) If $x(0) = \frac{1}{2}$, $p = q = 1$, solve the initial value problem for this equation and determine the limiting value of $x(t)$ as $t \to +\infty$.