MATH 2403, Spring 2013

Practice Exam 2, Chapters 3, 6 and 7

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. Calculator is not allowed in this exam. One side of a letter sized formula sheet is allowed. You have 50 minutes.

Problem 1 Consider the system of differential equations

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = \frac{x-ty}{1-t}. \end{cases} \tag{1}$$

(a) What are the restrictions on t_0, x_0 and y_0 in order that this system of equations has a unique solution for which $x(t_0) = x_0$ and $y(t_0) = y_0$.

(b) Find a solution $(x(t), y(t))$ of (1) for which $y(t) = 1$ is constant.

(c) Verify that $(x, y) = (e^t, e^t)$ is another solution of (1.

(d) Find a solution which satisfies the initial conditions $x(0) = 1$ and $y(0) = 0$.

Problem 2. Consider the predator-prey system

$$\begin{cases} \frac{dx}{dt} = 5x - xy \\ \frac{dy}{dt} = -2y + xy \end{cases}$$

(a) Find all critical points for this system.

(b) There is a critical point in first quadrant (if not, check your answer in (a)), find the corresponding linearization of your system near this critical point.

(c) Sketch the phase portraits for the linear system you found in part (b), indicating the type and the stability of the critical point for this linear system.

(d) By integrating $\frac{dy}{dx}$, find the equation of trajectory curves for the nonlinear system in the phase plane.

Problem 3 (18 points) Consider the system of equations

$$\begin{cases} \frac{dx}{dt} = -8x + 5y, \\ \frac{dy}{dt} = -10x + 7y, \end{cases} \tag{2}$$

(a) Find the general solution of (2).

(b) Sketch the solutions from (a) in the (x, y) plane, state the type and stability of the origin, indicate clearly in which direction t increases and the behavior for large t.

(c) From your sketch in part (b), what can you say about the long-term behavior of the solution with initial condition $x(0) = 5$, $y(0) = 11$?

(d) Let A be the coefficient matrix in system (2). Compute e^A.