Math 4305, Spring 2016,
Midterm 1, Practice

Show all your work. You may use one side of a sheet sized paper for formula in this exam. No calculator is allowed. Please give yourself 50 minutes.

Problem 1 Suppose that the matrix below is the augmented matix of a system of linear equations

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & -2 & 0 & 1 & 3 \\ 0 & 0 & 1 & k & h \end{pmatrix}$$

a) For what values of h and k, this system has no solution.

b) For what values of h and k, this system has a unique solution. Find the solution.

c) For what values of h and k, this system has infinitely many solutions. Describe the set of all solutions using parametric vector form.

Problem 2 Let $\mathbf{v} = (1, 0, 1)^t$. Define the linear transformation $T : \mathbf{R^3} \to \mathbf{R^3}$ by $T(\mathbf{x}) = \mathbf{v} \times x$. Where $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$.

a) Find the standard matrix A of T.

b) Find a basis of $im(A)$.

c) What's the dimension of $ker(A)$?

Problem 3 Consider an $m \times n$ matrix A and an $n \times m$ matrix B (with $n \neq m$) such that $AB = I_m$. Are the columns of B linearly independent? What about columns of A?

Problem 4 Let $S = \{(x, y) : xy \geq 0\}$ be a subset of the plane $\mathbf{R}^2$. Is S a subspace of $\mathbf{R}^2$?

Problem 5 For which values of the cosntant k is the following matrix invertible? Find the inverse.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{pmatrix}$$