MATH 2401, Summer 2014
Practice Exam 1, Chapter 13 and 14

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of an half of sheet of paper (A4) for formulas, calculator is NOT allowed in this exam. You have 70 minutes.

Problem 1. Calculations.

(a) $\frac{d}{dt}[(2t\mathbf{i} + \sqrt{t}\mathbf{j}) \bullet (t\mathbf{i} - 3\mathbf{j})]$

(b) $\frac{d}{dt}[(cost\mathbf{i} + sint\mathbf{j} + t\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})]$

(c) $\frac{d}{dt}[e^{cos2t}\mathbf{i} + ln(1 + t^2)\mathbf{j} + (1 - cost)\mathbf{k}]$

(d) Let $w = f(x, y, z) = xz + e^{y^2 z} + \sqrt{xy^2 z^3}$, calculate w_x, w_y, w_z, w_{xy} and w_{yz}

(e) Set $f(x, y) = \frac{x^2 - y^4}{x^3 - y^4}$. Determine whether or not f has a limit at $(1, 1)$.

(f) Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $P(1, -1, 1)$ in the direction of $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

(g) Find the rate of change of $f(x, y) = xe^y + ye^{-x}$ along the curve $\mathbf{r}(t) = (lnt)\mathbf{i} + t(lnt)\mathbf{j}$.

(h) Find $\frac{\partial u}{\partial s}$ for $u = x^2 - xy$, $x = scost$, $y = tsins$.

(i) Find $\frac{dy}{dx}$ if $xcos(xy) + ycos(x) = 2$.

(j) Is $\mathbf{F}(x, y) = (x + siny)\mathbf{i} + (xcosy - 2y)\mathbf{j}$ a gradient of a function $f(x, y)$? If yes, find the general form of $f(x, y)$.

Problem 2 A golf ball is hit at time $t = 0$. Its position vector as a function of time is given by

$$\mathbf{r}(t) = 2t\mathbf{i} + 3t\mathbf{j} + (-t^2 + 4t)\mathbf{k}.$$

Notice that at $t = 0$ the ball is at the origin of the coordinate system. The xy plane represents the ground. At some time $t_1 > 0$ the ball will return to the xy plane at some point $P(a, b, 0)$.

(a) Compurte the velocity, the accelaration and the speed of the ball at an arbitrary time t.

(b) Find the time $t_1 > 0$ and the coordinates of the point P where the ball hits the xy plane again.

(c) Set up a definite integral equal to the length of the arc of the trajectory from the origin to the point P. You do not have to evaluate the integral.

(d) Find the equation of the line tangent to the trajectory at P.

(e) Find the equation of the vertical plane containing the trajectory.

(f) Find the curvature of the trajectory at P.

Problem 3 At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force:

$\mathbf{F}(t) = m\pi^2[4cos(\pi t)\mathbf{i} + 3sin(\pi t)\mathbf{j}]$. Given that $\mathbf{v}(0) = -3\pi\mathbf{j} + \mathbf{k}$, and $\mathbf{r}(0) = 3\mathbf{j}$. find the following:

(a) The velocity $\mathbf{v}(1)$.

(b) The speed $v(1)$.

(c) The momentum $\mathbf{p}(1)$.

(d) The angular momentum $\mathbf{L}(1)$.

(e) The torque $\tau(1)$.

(f) The position $\mathbf{r}(1)$.

Problem 4 Consider the function $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$.

(a) Find the equation for the tangent plane to the level surface $f = 4$ at the point $P(1, 4, 1)$.

(b) Find the equation for the normal line to $f = 4$ at $P(1, 4, 1)$.

(c) Use differentials to estimate $f(0.9, 4.1, 1.1)$.

Problem 5. Find the area of the largest rectangle with edges parrallel to the coordinate axes that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Problem 6 Find the absolute extreme values taken on $f(x, y) = \frac{-2y}{x^2+y^2+1}$ on

the set $D = \{(x, y) : x^2 + y^2 \leq 4\}$.