MATH 2401, Summer 2014
Practice Final

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One sheet of paper (8.5 inch X 11.5 inch) for formulas (one side) is allowed. Calculator is not allowed in this exam. Try to work this practice within 150 minutes.

Problem 1. This problem is about the function

$$f(x, y, z) = 3zy + 4xcos(z).$$

(a) Find the rate of change of the function f at $(1, 1, 0)$ in the direction from this point to the origin.

(b) Give an approximate value of $f(0.9, 1.2, 0.11)$

(c) The equation $f(x, y, z) = 4$ implicitly defines z as a function of (x, y), if we agree that $z = 0$ if $(x, y) = (1, 1)$. Find the numnerical values of the derivatives:

$\frac{\partial z}{\partial x}(1, 1)$ and $\frac{\partial z}{\partial y}(1, 1)$.

(d) Suppose $\mathbf{r}(t) = (x(t), y(t), z(t))$ is a parametric curve such that $\mathbf{r}(0) =$

$(1,1,0)$ and $\mathbf{r}'(0) = (3,2,1)$. Find the value of

$$\frac{d}{dt} f(\mathbf{r}(t))|_{t=0}.$$

Problem 2 (a) Find the value of a such that the field on the plane

$$\mathbf{F}(x,y) = (axy)\mathbf{i} + x^2\mathbf{j}$$

is conservative. Find a potential for the resulting field.

(b) Compute the line integral of the conservative field you found in part (a) over the curve $\mathbf{r}(t) = e^{t^2}\mathbf{i} + t cos(2\pi t)\mathbf{j}$, $0 \leq t \leq 1$.

Problem 3. Evaluate $I = \displaystyle\int_{C_R} dx + x^2 y dy$, where C_R is the triangle with vertices $(0,0)$, $(0,R)$, $(R,0)$ oriented counterclockwise.

Problem 4 Let S be the portion of the surface $x = 5 - y^2 - z^2$ in the half space $x \geq 1$, oriented so that the normal vector at $(5,0,0)$ is equal to $\mathbf{i}$. Let $\mathbf{F}(x,y,z) = -\mathbf{i} + \mathbf{j}$ (a constant vector field).

(a) Set up and evaluate the flux of $\mathbf{F}$ across S.

(b) Verify that $\mathbf{F} = \nabla \times \mathbf{G}$, where $\mathbf{G} = z\mathbf{j} - x\mathbf{k}$.

(c) Give an alternative calculation of the surface integral of part (a) by applying Stokes' theorem.

Problem 5 For each item, circle the correct answer or indicate if the statement is ture or false. Assume that the functions, fields and curves below are smooth.

(a) Let C be an arc from $(0,0)$ to $(2,1)$. According to the fundamental theorem for line integrals, $\int_C (y-1)dx + (x+2y)dy$ is equal to

(1) 2, (2) 1, (3) It depends on what C is.

(b) For every smooth function f, the integral $\int_0^1 \int_0^{2y^2+1} f(x,y)\ dxdy$ is equal to

(1) $\int_0^3 \int_0^{\sqrt{\frac{1}{2}(x-1)}} f(x,y)\ dydx$,

(2) $\int_1^3 \int_0^{\sqrt{\frac{1}{2}(x+1)}} f(x,y)\ dydx$,

(3) None of the above.

(c) If $\mathbf{F}$ is a field such that $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$ where C is the unit circle, then $\mathbf{F}$ must be conservative.

(1) True, (2) False.

(d) If C is the boundary of a bounded domain D and C is oriented as in the statement of Green's theorem, then $\int_C x^2 y dx - y dy$ equals

(1) $\int\int_D (2xy - 1) dxdy$

(2) $\int\int_D (1 - x^2) dxdy$

(3) $\int\int_D (-x^2) dxdy$

(4) None of the above.

(e) If (a, b) is a critical point of a function f, and if

$$f_{xx}(a, b) = -2, \ and \ f_{yy}(a, b) = 1,$$

then what can one say about (a, b)?

(1) Noting can be concluded from the given information.

(2) (a, b) is a local minimum of f

(3) (a, b) is a local maximum of f

(4) (a, b) is a saddle point of f

Problem 6 Consider the surface S that is the part of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$.

(a) Give a parametirc representation of S. Make sure to explicitly describe or sketch the parametrization domain D.

(b) Find an equation of the tangent plane to S at the point $P(-1, 1, \sqrt{2})$.

(c) If the density function $\lambda(x, y, z)$ is equal to the distance to the xy-plane, find the total mass of the surface S.

Problem 7 Let E denote the portion of the solid ball of radius R in the first octant, and let

$$\mathbf{F} = (2x + y)\mathbf{i} + y^2\mathbf{j} + cos(xy)\mathbf{k}.$$

Applying the Divergence Theorem, compute the net flux of the field $\mathbf{F}$ across the boundary of E, oriented by the outward-pointing normal vectors.

Problem 8 Please complete the course survey. Your comments will help me to improve my teaching in the future. Thank you in advance.