

**Math 4305, Summer 2015,
Midterm, Practice**

Show all your work. Please give yourself 100 minutes.

Problem 1 Suppose that the matrix below is the augmented matrix of a system of linear equations

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & -2 & 0 & 1 & 3 \\ 0 & 0 & 1 & k & h \end{array} \right)$$

- a) For what values of h and k , this system has no solution.
- b) For what values of h and k , this system has a unique solution. Find the solution.
- c) For what values of h and k , this system has infinitely many solutions. Describe the set of all solutions using parametric vector form.

Problem 2 Let $\mathbf{v} = (1, 0, 1)^t$. Define the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$

by $T(\mathbf{x}) = \mathbf{v} \times \mathbf{x}$. Where $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$.

- a) Find the standard matrix A of T .
- b) Solve $A\mathbf{x} = \mathbf{0}$.

Problem 3 For which values of the constant k is the following matrix invertible? Find the inverse.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & 4 & k^2 \end{pmatrix}$$

Problem 4 Consider an $n \times m$ matrix A of rank n . Show that there exists an $m \times n$ matrix X such $AX = I_n$. If $n < m$, how many such matrices X are there?