MATH 4305, Summer 2015,
Exam 2, Practice

Show all your work. Please give yourself 100 minutes.

Problem 1 Let $\mathbf{v} = (1,0,1)^t$. Define the linear transformation $T : \mathbf{R^3} \to \mathbf{R^3}$ by $T(\mathbf{x}) = \mathbf{v} \times x$. Where $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$.

a) Find the standard matrix A of T.

b) Find a basis of $im(A)$.

c) What's the dimension of $ker(A)$?

Problem 2 Consider an $m \times n$ matrix A and an $n \times m$ matrix B (with $n \neq m$) such that $AB = I_m$. Are the columns of B linearly independent? What about columns of A?

Problem 3 Find a basis $\mathbf{B}$ of $\mathbf{R}^2$ such that

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{\mathbf{B}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \qquad \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{\mathbf{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Problem 4 Find all possible values of a so that the columns of A given below are linearly dependent?

$$\begin{pmatrix} a & 2a & 0 & 0 \\ 0 & 0 & a-3 & 3(a-3) \\ 0 & -2a & 0 & 1 \\ 0 & 0 & a-2 & 2(a-2) \end{pmatrix}$$

Problem 5 (a) Prove that the set $\mathbf{B} = \{1+t^2, t+t^2, 1+2t+t^2\}$ is a basis for $\mathbf{P}_2$.

b) Find the matrix of the linear transformation $T(f(t)) = f' - 3f$ from $\mathbf{P}_2$ to $\mathbf{P}_2$ withe respect to the basis $\mathbf{B}$ found in part (a).