Large time behavior of Euler-Poisson system for semiconductor

Feimin Huang[1] [*], **Ronghua Pan**[2] [†], **Huimin Yu**[1] [‡]

[1]Institute of Applied Mathematics, AMSS, Academia Sinica, Beijing 100080, China
[2]School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332, USA

Abstract: In this note, we present a framework for the large time behavior of general uniformly bounded weak entropy solutions to the Cauchy problem of Euler-Poisson system of semiconductor devices. It is shown that the solutions converges to the stationary solutions exponentially in time. No smallness and regularity conditions are assumed.

1 Introduction

Consider the following one-dimensional Euler-Poisson system modeling semiconductor devices:

$$\begin{cases} n_t + J_x = 0, \\ J_t + \left(\frac{J^2}{n} + p(n)\right)_x = nE - J, \qquad x \in \mathbf{R},\ t > 0, \\ E_x = n - b(x). \end{cases} \tag{1}$$

Here $n \geq 0$, J, and E denote the electron density, electron current density and the (negative) electric field, respectively. The function $b = b(x) > 0$, called doping profile, stands for the density of fixed, positively charged background ions. In this paper, we assume $b(x)$ satisfies

$$\begin{aligned} b(x) \in C^2(\mathbf{R}),\ b'(x) \in L^1(\mathbf{R}) \cap H^1(\mathbf{R}), \\ \lim_{x\to\pm\infty} b(x) = b^{\pm} > 0, \quad b^* = \sup_{x\in\mathbf{R}} b(x) \geq \inf_{x\in\mathbf{R}} b(x) = b_* > 0. \end{aligned} \tag{2}$$

*Email address: fhuang@amt.ac.cn,
†Email address: panrh@math.gatech.edu,
‡Email address: hmyu@amss.ac.cn,

We also assume the pressure p satisfies the γ-law: $p(n) = n^\gamma$ ($\gamma \geq 1$). Several physical constants have been set to unity for the simplicity of presentation. Such a system, replacing the most commonly used drift-diffusion model for charged carriers, is valid in the region occupied by the semiconductor. We refer to [14] for background on modeling and analysis.

This system is supplemented with a condition at $x = -\infty$ for the electric field

$$\lim_{x \to -\infty} E(x,t) = E^-, \qquad for \quad a.e. \quad t \in [0, +\infty), \tag{3}$$

and the initial conditions

$$n(x,0) = n_0(x), \qquad J(x,0) = J_0(x) \tag{4}$$

such that

$$\lim_{x \to \pm\infty} n_0(x) = b^\pm > 0, \qquad \lim_{x \to \pm\infty} J_0(x) = \bar{J}, \quad b_- E^- = \bar{J}. \tag{5}$$

Due to the relaxation mechanism, smooth solutions exist globally in time if the initial data is chosen from the so-called sub-critical region; see [1] and [17]. However, when data is out of that region, the solutions break down in finite time, c.f. [2]. The global existence of weak entropy solutions are proven in [13], [16] and [18]. Concerning with the large time behavior of solutions, we refer to [10], [11] for small smooth solutions, and to [9] for piecewise smooth solutions. There were also some results concerning the initial boundary value problems, we refer the readers to [8], [12] and [4] and the references therein.

In this paper, we will focus on a framework on large time asymptotic behavior that applies to any uniformly bounded entropy weak solutions. The entropy and entropy flux pair we will use here are the physical ones defined as following:

$$\begin{aligned} \eta_e &= \frac{J^2}{2n} + \frac{n^\gamma}{\gamma - 1}, \qquad q_e = \frac{J^3}{2n^2} + \frac{\gamma}{\gamma - 1} n^{\gamma-1} J, \quad \gamma > 1; \\ \eta_e &= \frac{J^2}{2n} + n \ln n, \qquad q_e = \frac{J^3}{2n^2} + (\ln n + 1) J, \quad \gamma = 1. \end{aligned} \tag{6}$$

We now define the concept of entropy weak solutions.

Definition 1: The bounded measurable function $(n, J, E)(x,t)$ is said to be an entropy weak solution of problem (1), (3)–(5), if it satisfies the system

(1) in the distributional sense, verifies the initial and limiting restrictions (3)–(5), and the following entropy inequality

$$\eta_{et} + q_{ex} + \frac{J}{n}(J - nE) \leq 0, \tag{7}$$

holds in the distributional sense.

Throughout this paper, we assume

H1): Assume that $(n, J, E)(x, t)$ is any globally defined weak entropy solutions which satisfies

$$0 \leq n(x, t) \leq C_0. \tag{8}$$

Under this assumption, we will prove that the entropy weak solution defined above converges exponentially fast toward the corresponding stationary states if the background current $\bar{J}$ has small amplitude. In section 2, we will study the stationary states in subsonic region. Our main result will be established in section 3. Some remarks are collected in section 4.

2 Stationary states

Again, due to the relaxation mechanism in the current equation of (1), we expect that all the solutions of our problem converge to the solutions of the following stationary problem,

$$\left\{ \begin{array}{l} \tilde{J}_x = 0, \\ (\frac{\tilde{J}^2}{\tilde{n}} + p(\tilde{n}))_x = \tilde{n}\tilde{E} - \tilde{J}, \\ \tilde{E}_x = \tilde{n} - b(x), \end{array} \right. \tag{9}$$

under the conditions

$$\tilde{n}(x) - b(x) \in H^1(\mathbf{R}), \quad \tilde{J} = \bar{J}, \quad \tilde{E}(-\infty) = E^-. \tag{10}$$

A straightforward calculation (see [9]) shows that (9)–(10) gives

$$[(\frac{p'(\tilde{n})}{\tilde{n}} - \frac{\bar{J}^2}{\tilde{n}^3})\tilde{n}_x]_x + (\frac{\bar{J}}{\tilde{n}})_x = \tilde{n} - b(x), \tag{11}$$

which is a second order ODE for $\tilde{n}(x)$. Clearly, the strictly elliptic condition is equivalent to

$$p'(\tilde{n}) > \frac{\bar{J}^2}{\tilde{n}^2}. \tag{12}$$

This is exactly the subsonic condition [3], recalling that $\frac{J}{n}$ represents the particle velocity. We also note that (12) is equivalent to

$$\tilde{n} > (\frac{\bar{J}^2}{\gamma})^{\frac{1}{(\gamma+1)}}. \tag{13}$$

In order to ensure the subsonic condition, we assume that

H2): $b(x)$ and $\bar{J}$ satisfy $b_* > (\frac{\bar{J}^2}{\gamma})^{1/(\gamma+1)}$.

Under **H2)**, it is proven by [11] (see also [9]) that

Theorem 1: Suppose $b(x)$ satisfies the condition (2). Assume that **H2)** holds. Then problem (9)-(10) has a unique solution $(\tilde{n}, \bar{J}, \tilde{E})$, such that $b_\pm \tilde{E}(\pm\infty) = \bar{J}$, and

$$b_* \leq \tilde{n}(x) \leq b^*, \quad x \in \mathbf{R}, \tag{14}$$

$$|\tilde{n}(x) - b_\pm| = O(1)e^{-c_\pm |x|}, \quad as \quad x \to \pm\infty, \tag{15}$$

$$\|\tilde{n} - b\|_{H^2} + \sup_{x\in\mathbf{R}}(|\tilde{n}'(x)| + |\tilde{n}''(x)| + |\tilde{E}(x)|) \leq C_1, \tag{16}$$

where C_1 is a positive constant that only depends on $b(x)$, and

$$c_\pm = \frac{\tilde{E}_\pm}{p'(b_\pm) - \tilde{E}_\pm^2}.$$

We remark that, (14) and **H2)** together ensure the subsonic condition (13) and then the proof of Theorem 1 is carried out by standard ODE theory. The proof of statement (14) is done through a comparison argument as that in [10].

3 Large time behavior

Now, our aim is to prove the entropy-weak solution of (1), (3)-(5) strongly converges to its stationary solution in $L^2(\mathbf{R})$ with exponential decay rate. We set

$$y = -(E - \tilde{E}). \tag{17}$$

Then systems (1) and (9) infer that

$$y_x = -(n - \tilde{n}), \qquad y_t = J - \bar{J}. \tag{18}$$

As expected, the entropy inequality (7) will play an important role in our analysis. For this purpose, we introduce

$$\begin{aligned} \eta_* &= \eta_e - \tilde{\eta}_e - \nabla\tilde{\eta}_e(\overrightarrow{v} - \overrightarrow{\tilde{v}}), \\ q_* &= q_e - \tilde{q}_e - \nabla\tilde{\eta}_e(\overrightarrow{f} - \overrightarrow{\tilde{f}}), \end{aligned} \tag{19}$$

where

$$\begin{aligned} &\tilde{\eta}_e = \eta_e(\tilde{n}, \bar{J}), \qquad \tilde{q}_e = q_e(\tilde{n}, \bar{J}), \\ &\overrightarrow{v} = (n, J)^T, \qquad \overrightarrow{f} = (J, \frac{J^2}{n} + p(n))^T. \end{aligned} \tag{20}$$

The following theorem is our main result.

Theorem 2: Let $(\tilde{n}, \bar{J}, \tilde{E})$ be given in Theorem 1. Let $(n, J, E)(x, t)$ be any weak entropy solution of (1), (3)–(5) satisfying **H1)** such that $y(x, 0) \in L^2(\mathbf{R})$, $\int_{-\infty}^{\infty} \eta_*(x, 0)dx < \infty$, then there exists a positive constant δ such that if

$$\bar{J} \leq \delta,$$

then

$$\int_{-\infty}^{\infty} (y_t^2 + y_x^2 + y^2)dx \leq Ce^{-\tilde{C}t} \int_{-\infty}^{\infty} (\eta_*(x, 0) + y^2(x, 0))\, dx, \tag{21}$$

holds for any $t > 0$ and some positive constants C and $\tilde{C}$.

Proof: First, from (17) and (18) we have the following equation on y

$$y_{tt} + (\frac{J^2}{n} - \frac{\bar{J}^2}{\tilde{n}})_x + (p(n) - p(\tilde{n}))_x + y_t = -\tilde{n}y - \tilde{E}y_x + yy_x. \tag{22}$$

Multiplying y with (22) and integrating over $(-\infty, +\infty)$, we get

$$\begin{aligned}\frac{d}{dt}\int_{-\infty}^{\infty}(yy_t+\frac{1}{2}y^2)dx+\int_{-\infty}^{\infty}(p(n)-p(\tilde{n}))(n-\tilde{n})+(\tilde{n}-\frac{\tilde{E}_x}{2})y^2)\ dx\\ \le\int_{-\infty}^{\infty}y_t^2dx+\int_{-\infty}^{\infty}(\frac{J^2}{n}-\frac{\bar{J}^2}{\tilde{n}})y_x\ dx.\end{aligned} \tag{23}$$

We now note that

$$\tilde{n}-\frac{\tilde{E}_x}{2}=\frac{1}{2}(\tilde{n}+b(x)). \tag{24}$$

Thanks to (14), the Lemma 3.1 of [6] says that

$$(p(n)-p(\tilde{n}))(n-\tilde{n})=O(1)(n-\tilde{n})^2=O(1)y_x^2. \tag{25}$$

Therefore, we conclude from (23) that there exists a positive constant C_2 such that

$$\begin{aligned}\frac{d}{dt}\int_{-\infty}^{\infty}(yy_t+\frac{1}{2}y^2)dx+C_2\int_{-\infty}^{\infty}(y_x^2+y^2)\ dx\\ \le\int_{-\infty}^{\infty}y_t^2dx+\int_{-\infty}^{\infty}(\frac{J^2}{n}-\frac{\bar{J}^2}{\tilde{n}})y_x\ dx.\end{aligned} \tag{26}$$

As our solution has no further regularity, standard energy estimates such as (26) do not generalize to higher order. Following the ideas introduced in [5] and [6], we will now explore the entropy dissipation. In view of the definition of η_* and q_* in (19), we substitute them into the entropy inequality (7) to obtain

$$\begin{aligned}0\ge\ &\eta_{et}+q_{ex}-\frac{J}{n}(nE-J)\\ &=\eta_{*t}+q_{*x}+\nabla\tilde{\eta}_e(\overrightarrow{v}-\overrightarrow{\tilde{v}})_t+\nabla\tilde{\eta}_e(\overrightarrow{f}-\overrightarrow{\tilde{f}})_x\\ &\quad+\tilde{q}_{ex}+(\nabla\tilde{\eta}_e)_x(\overrightarrow{f}-\overrightarrow{\tilde{f}})-\frac{J}{n}(nE-J).\end{aligned} \tag{27}$$

We now simplify the terms in (27). Using the equations (17), (18) and (22), we observe that

$$\nabla\tilde{\eta}_e(\overrightarrow{v}-\overrightarrow{\tilde{v}})_t+\nabla\tilde{\eta}_e(\overrightarrow{f}-\overrightarrow{\tilde{f}})_x=\frac{\bar{J}}{\tilde{n}}(nE-\tilde{n}\tilde{E}-y_t), \tag{28}$$

and

$$\begin{aligned}&\tilde{q}_{ex}+(\nabla\tilde{\eta}_e)_x(\overrightarrow{f}-\overrightarrow{\tilde{f}})\\ &=[\gamma\tilde{n}^{\gamma-2}J+\frac{\bar{J}^2}{\tilde{n}^3}J-2\frac{\bar{J}^3}{\tilde{n}^3}-\frac{\bar{J}}{\tilde{n}^2}(\frac{J^2}{n}-\frac{\bar{J}^2}{\tilde{n}}+p(n)-p(\tilde{n}))]\tilde{n}_x.\end{aligned} \tag{29}$$

We thus have from (27) that

$$\begin{aligned}\eta_{*t}+q_{*x} &\le JE-\frac{J^2}{n}-\frac{\bar J}{\tilde n}(nE-\tilde n\tilde E-y_t)\\ &\quad -[\tilde q_{ex}+(\nabla\tilde\eta_e)_x(\overrightarrow{f}-\overrightarrow{\bar f})].\end{aligned}\tag{30}$$

For further simplification is necessary, we note that

$$\tilde E=\frac{1}{\tilde n}(\frac{\bar J^2}{\tilde n}+p(\tilde n))_x+\frac{\bar J}{\tilde n}=\gamma\tilde n^{\gamma-2}\tilde n_x-\frac{\bar J^2}{\tilde n^3}\tilde n_x+\frac{\bar J}{\tilde n},$$

then

$$\begin{aligned}JE &= J\tilde E-yy_t-y\bar J\\ &= -\frac{\bar J^2}{\tilde n^3}J\tilde n_x+\gamma\tilde n^{\gamma-2}J\tilde n_x+\frac{\bar J}{\tilde n}y_t+\frac{\bar J^2}{\tilde n}-yy_t-y\bar J.\end{aligned}\tag{31}$$

On the other hand, we have

$$\begin{aligned}&-\frac{\bar J}{\tilde n}(nE-\tilde n\tilde E-y_t)=-\frac{\bar J}{\tilde n}(yy_x-\tilde Ey_x-y\tilde n-y_t)\\ =\;&-\frac{\bar J^3}{\tilde n^4}y_x\tilde n_x+\gamma\tilde n^{\gamma-3}\bar Jy_x\tilde n_x+\frac{\bar J^2}{\tilde n^2}y_x-\frac{\bar J}{\tilde n}yy_x+\bar Jy+\frac{\bar J}{\tilde n}y_t.\end{aligned}\tag{32}$$

From (29)–(32), manipulating the terms properly, we conclude that

$$(\eta_*+\frac{1}{2}y^2)_t+q_{*x}+Q_1\le\frac{\bar J\tilde n_x}{\tilde n^2}(Q_1+Q_2+\frac{1}{2}y^2)-(\frac{\bar J}{2\tilde n}y^2)_x,\tag{33}$$

where

$$Q_1=\frac{J^2}{n}-\frac{\bar J^2}{\tilde n}-\frac{2\bar J}{\tilde n}y_t-\frac{\bar J^2}{\tilde n^2}y_x,\quad Q_2=n^\gamma-\tilde n^\gamma+\gamma\tilde n^{\gamma-1}y_x.\tag{34}$$

Clearly, Q_1 is the quadratic remainder of the Taylor expansion of $\dfrac{J^2}{n}$ around $\bar J$ and $\tilde n$, while Q_2 is the one for pressure. Furthermore, $Q_2=0$ for $\gamma=1$. Due to the convexity, we remark that both Q_1 and Q_2 are non-negative.

Integrating (33) over $(-\infty,+\infty)$, we get

$$\frac{d}{dt}\int_{-\infty}^{\infty}(\eta_*+\frac{1}{2}y^2)dx+\int_{-\infty}^{\infty}Q_1dx\le\int_{-\infty}^{\infty}\frac{\bar J\tilde n_x}{\tilde n^2}(Q_1+Q_2+\frac{1}{2}y^2)dx.\tag{35}$$

Let $\Lambda = \max\{b^*, C_0\}$, where C_0 is given in **H1)**. We now multiply (35) by $\lambda = 2\Lambda + 1$, add the results to (23) to obtain

$$\frac{d}{dt}\int_{-\infty}^{\infty} F_1\ dx + \int_{-\infty}^{\infty}(F_2 + F_3 + F_4)\ dx \le 0, \tag{36}$$

where

$$\begin{aligned} F_1 &= \lambda\eta_* + \frac{1}{2}(1+\lambda)y^2 + yy_t, \\ F_2 &= \lambda(1 - \frac{\bar{J}\tilde{n}_x}{\tilde{n}^2})Q_1 - y_t^2 - (\frac{J^2}{n} - \frac{\bar{J}^2}{\tilde{n}})y_x, \\ F_3 &= (p(n) - p(\tilde{n}))(n - \tilde{n}) - \lambda\frac{\bar{J}\tilde{n}_x}{\tilde{n}^2}Q_2, \\ F_4 &= [(\tilde{n} - \frac{\tilde{E}_x}{2}) - \frac{1}{2}\lambda\frac{\bar{J}\tilde{n}_x}{\tilde{n}^2}]y^2. \end{aligned} \tag{37}$$

By Theorem 1, we know there is $C_3 > 0$ such that

$$\frac{\bar{J}\tilde{n}_x}{\tilde{n}^2} \le C_3\delta.$$

For F_4, it is clear from (14) and (24) that

$$F_4 \ge \frac{1}{2}(2b_* - \lambda C_3\delta)y^2. \tag{38}$$

When $\gamma = 1$, $Q_2 = 0$, and thus $F_3 = y_x^2$. When $\gamma > 1$, Lemma 5.2 of [15] states that there exits a positive C_4 such that

$$Q_2 \le C_4(p(n) - p(\tilde{n}))(n - \tilde{n}),$$

and therefore

$$F_3 \ge (1 - C_3C_4\lambda\delta)(p(n) - p(\tilde{n}))(n - \tilde{n}). \tag{39}$$

We now treat F_2. From the definition of Q_1 in (34), we know that

$$(\frac{J^2}{n} - \frac{\bar{J}^2}{\tilde{n}})y_x = y_xQ_1 + \frac{2\bar{J}}{\tilde{n}}y_ty_x + \frac{\bar{J}^2}{\tilde{n}^2}y_x^2. \tag{40}$$

On the other hand,

$$\begin{aligned} Q_1 &= \frac{J^2}{n} - \frac{J^2}{\tilde{n}} + \frac{J^2}{\tilde{n}} - \frac{\bar{J}^2}{\tilde{n}} - \frac{2\bar{J}}{\tilde{n}}y_t - \frac{\bar{J}^2}{\tilde{n}^2}y_x \\ &= \frac{y_t^2}{\tilde{n}} + \frac{y_x}{\tilde{n}}(\frac{J^2}{n} - \frac{\bar{J}^2}{\tilde{n}}) \\ &= \frac{y_t^2}{\tilde{n}} + \frac{y_x}{\tilde{n}}(Q_1 + \frac{2\bar{J}}{\tilde{n}}y_t + \frac{\bar{J}^2}{\tilde{n}^2}y_x), \end{aligned}$$

from which we have

$$\frac{n}{\tilde{n}}Q_1 = \frac{y_t^2}{\tilde{n}} + \frac{2\bar{J}}{\tilde{n}^2}y_t y_x + \frac{\bar{J}^2}{\tilde{n}^3}y_x^2. \tag{41}$$

Then we get

$$\begin{cases} Q_1 \geq \dfrac{y_t^2}{\tilde{n}} + \dfrac{2\bar{J}}{\tilde{n}^2}y_t y_x, \quad n \leq \tilde{n}; \\ Q_1 \geq \dfrac{\tilde{n}}{n}(\dfrac{y_t^2}{\tilde{n}} + \dfrac{2\bar{J}}{\tilde{n}^2}y_t y_x), \quad n > \tilde{n}. \end{cases} \tag{42}$$

We are now able to give a good estimate on F_2. From (40) and (42), and the fact that $|y_x| \leq \Lambda$, we have, for some positive constants C_5 and C_6, that

$$\begin{aligned} F_2 &\geq [\lambda(1 - C_3\delta) - y_x]Q_1 - y_t^2 - \frac{2\bar{J}}{\tilde{n}}y_t y_x - \frac{\bar{J}^2}{\tilde{n}^2}y_x^2 \\ &\geq C_5 Q_1 - C_6\delta y_x^2, \end{aligned} \tag{43}$$

provided $C_3\lambda\delta < \frac{1}{2}$, this is achieved if $\delta < \frac{1}{2}(C_3\lambda)^{-1}$. Therefore, for suitably small δ, we have from (38), (39) and (43) that there is $C_7 > 0$ such that

$$F_2 + F_3 + F_4 \geq C_7(Q_1 + y_x^2 + y^2). \tag{44}$$

We now turn to F_1. It is easy to see that

$$\eta_* = \frac{Q_1}{2} + \frac{1}{\gamma - 1}Q_2, \quad \gamma > 1; \qquad \eta_* = \frac{Q_1}{2} + Q_3, \quad \gamma = 1, \tag{45}$$

where

$$Q_3 = n\ln n - \tilde{n}\ln\tilde{n} - (\ln\tilde{n} + 1)(n - \tilde{n}). \tag{46}$$

From Lemma 3.1 of [6], we know that there is $C_8 > 0$ such that

$$Q_2 \geq C_8 y_x^2. \tag{47}$$

We now claim that there are $C_9 > 0$ and $C_{10} > 0$ such that

$$C_9 y_x^2 \leq Q_3 \leq C_{10} y_x^2. \tag{48}$$

Indeed, Q_3 is the quadratic remainder of the Taylor expansion of the convex function $n\ln n$ about $\tilde{n} \geq b_* > 0$. (48) is easily proven using the strict convexity of $n\ln n$ and the bound of n. Therefore, we conclude from (37), (42), (45)–(48) and the smallness of δ that there are $C_{11} > 0$ and $C_{12} > 0$ such that

$$C_{11}(y^2 + y_x^2 + y_t^2) \le C_{12}(y^2 + y_x^2 + Q_1) \le F_1 \le C_{13}(y^2 + y_x^2 + Q_1). \quad (49)$$

Hence, (36), (44) and (49) imply that

$$\frac{d}{dt}\int_{-\infty}^{\infty} F_1 \, dx + \frac{C_7}{C_{13}}\int_{-\infty}^{\infty} F_1 \, dx \le 0, \quad (50)$$

which together with (49) yields the decay estimate (21). This completes the proof of Theorem 2.

4 Conclusion Remarks

In this section, we make some remarks on our assumptions, results and proofs.

1) In the proof of Theorem 2, we used the fact that $y(\pm\infty, t) = 0$. The initial assumption does imply that $y(\pm\infty, 0) = 0$. Therefore, (21) and standard continuity argument do verify this fact.

2) We also remark that the smallness assumption on $\bar{J}$ also ensures **H2)**, that guarantees the subsonic condition. However, it is clear that the proof of Theorem 2 is still valid if we request small background velocity $\frac{\bar{J}}{b_*}$ instead. Furthermore, δ does not have to be arbitrary small in our Theorem 2. It is clear from our proof that one may determine a constant upper bound for δ depending on b_*, b^*, C_0 and certain convex functions appear in the proof. This will leads to a tedious elementary calculation which is not the main purpose of this paper. It is not clear whether the statement (21) is true without any restriction on the amplitude of $\bar{J}$ or background velocity.

3) The uniform upper bound on density is very important in our proof. We remark that the uniform bound (8) on n is still an open problem for L^∞ weak entropy solutions to system (1), although it seems natural from physical point of view. The bounds obtained in [13] or [16] grow in time. However, for the piecewise smooth solutions constructed in [9], the uniform bound (8) is verified. Also, the sub-critical global smooth solutions constructed in [17]

have uniform upper bound on n. Therefore, our Theorem 2 is valid for the solutions obtained by [9] and [17].

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